Design Problem 1

ME 3180-B
Group 3

Instructor
Dr. David "Alpha" Smith

Date of Submission
October 6, 2023

Team Members:
Skyy DuBose
Patrick Ferris Jr.
Cordell Palmer
Helen Snyder

DETERMINING THE LOADS

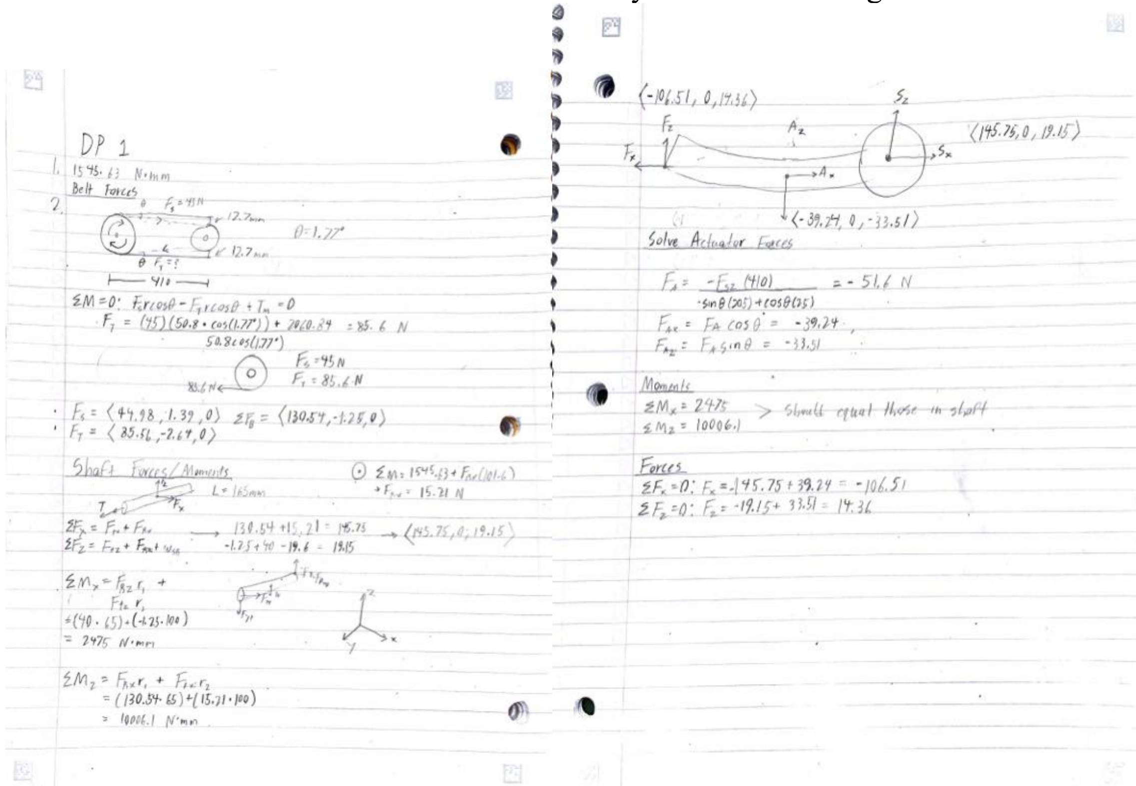
1. What is the blade torque required by these specifications?

$$T = \frac{9.55H}{n}, \text{ where}$$

H= Power in Watts and n=revolutions per minute

$$T = \frac{9.55(745.7)}{4600} = 1.54\ Nm \rightarrow 1540\ Nmm$$

2. There are many forces, moments, and reactions acting on the pivot arm. Provide a table of the locations (coordinates) of all forces and moments on the system that affect the design of the pivot arm, along with the vector components of the corresponding forces and moments. Use the world coordinate system shown in Figure 3.



DP 1

1. 1545.63 N·mm

Belt Forces

2.

θ $F_s = 45N$

12.7mm

$\theta = 1.77°$

12.7mm

θ $F_T = ?$

410

$\Sigma M = 0: F_s r\cos\theta - F_T r\cos\theta + T_m = 0$

$F_T = \frac{(45)(50.8 \cdot \cos(1.77°)) + 2060.84}{50.8\cos(1.77°)} = 85.6\ N$

85.6 N

$F_s = 45 N$

$F_T = 85.6 N$

$F_s = \langle 44.98, 1.39, 0\rangle$ $\Sigma F_B = \langle 130.54, -1.25, 0\rangle$

$F_T = \langle 85.56, -2.64, 0\rangle$

Shaft Forces/Moments

L = 165mm

$\Sigma M = 1545.63 + F_{tx}(101.6)$

$F_{tx} = 15.21\ N$

$\Sigma F_x = F_{Bx} + F_{tx}$ → 130.54 + 15.21 = 145.75 → $\langle 145.75, 0, 19.15\rangle$

$\Sigma F_z = F_{Bz} + F_{tz} + W_{sh}$ → -1.25 + 40 - 19.6 = 19.15

$\Sigma M_x = F_{Bz} r_1 + F_{tz} r_1$

$= (40 \cdot 65) + (-1.25 \cdot 100)$

$= 2475$ N·mm

$\Sigma M_z = F_{Bx} r_1 + F_{tx} r_2$

$= (130.54 \cdot 65) + (15.21 \cdot 100)$

$= 10006.1$ N·mm

$\langle -106.51, 0, 14.36\rangle$

$\langle 145.75, 0, 19.15\rangle$

$\langle -39.24, 0, -33.51\rangle$

Solve Actuator Forces

$F_A = \frac{-F_{Sz}(410)}{\sin\theta(205) + \cos\theta(25)} = -51.6\ N$

$F_{Ax} = F_A\cos\theta = -39.24$

$F_{Az} = F_A\sin\theta = -33.51$

Moments

$\Sigma M_x = 2475$

$\Sigma M_z = 10006.1$ > Should equal those in shaft

Forces

$\Sigma F_x = 0: F_x = -145.75 + 39.24 = -106.51$

$\Sigma F_z = 0: F_z = -19.15 + 33.51 = 14.36$

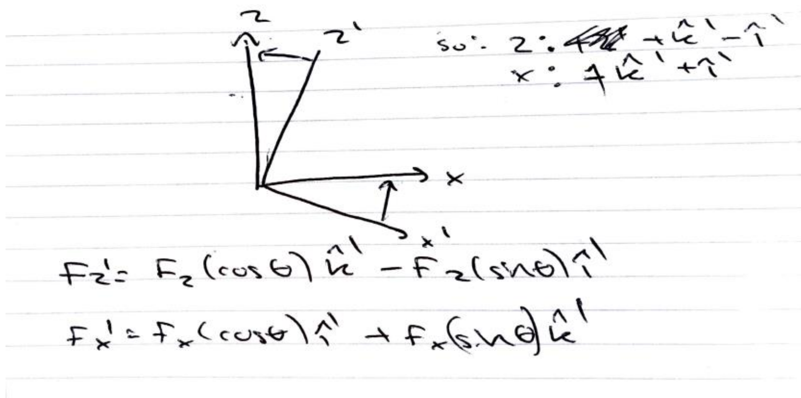
Note: There are a couple of sign errors or miswritten vectors, but all is correct in the table.

Forces	**Location**	**Vector Component**
Tension force in slack side of belt	(401, -100, -38.1)	$45\hat{\imath}$ (N)
Tension force in drive side of belt	(401, -100, 38.1)	$-85.6\hat{\imath}$ (N)
Shaft Force	(410, 0, 0)	$145.75\hat{\imath} + 19.15\hat{k}$ (N)
Moment about Shaft	(410, 0,0)	$2475\hat{\imath} + 10006.1\hat{k}$ (Nmm)
Actuator Force	(205, 0, -25)	$39.24\hat{\imath} + -33.51\hat{k}$ (N)
Weight of Shaft	(410, 0, 0)	$2\hat{k}$ (kg)
Weight of Pivot Arm	(205, 0, 0)	$-37.62\hat{k}$
Cutting Tangent Force	(410, 65, -101.6)	15.21k
Vertical Cutting Force	(410, 65, -101.6)	40j

3. In that same world coordinate system, what are the reaction forces and moments acting at the centerline of the pivot pin and actuator pin?

Forces	Location	Vector Component
Pivot Pin Forces	(0, 0, 0)	$-106.51\hat{\imath} + 14.36\hat{k}$
Moment at Pivot Pin	(0, 0, 0)	$2726\hat{\imath} + -12993\hat{\jmath} + 12064\hat{k}$ (N)
Actuator Pin Forces	(205, 0, 25)	39.227î + -33.503$\hat{k}$ (N)

4. In a local coordinate system (x′,y′,z′), rotated so that x′ is tangent to the curved pivot arm, y′ is still the pivot pin axis, and z′ is defined by those two axes according to the right hand rule, what are the equivalent forces and moments acting at the origin?



Forces	Vector Component	Local Coordinates
Tension force in slack side of belt	$45\hat{\imath}$ (N)	$44.6\hat{\imath}' + 5.445\hat{k}'$
Tension force in drive side of belt	-85.6$\hat{\imath}$ (N)	85.971$\hat{\imath}$‘ +10.358$\hat{k}$’
Shaft Force	145.75$\hat{\imath}$ + 19.15$\hat{k}$ (N)	142.064$\hat{\imath}'$ + 36.645$\hat{k}$’ (N)
Moment about Shaft	2475$\hat{\imath}$ + 10006.1$\hat{k}$ (Nmm)	
Actuator Force	-39.24$\hat{\imath}$ + -33.51$\hat{k}$ (N)	43.007$\hat{\imath}$ + -28.516$\hat{k}$ (N)
Weight of Shaft	2 (kg)	.242î’+-1.9853k’ (N)
Weight of Pivot Arm	-6.95645$\hat{k}$	.841î’+ -6.905k’(N)

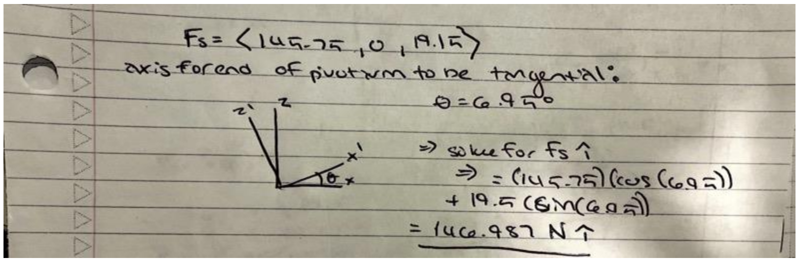
5. In the world coordinate system, what are the loads at each pin at each side of the pivot arm that the pin passes through in this design?

    At the pivot pin, F_x= -106.51 and F_z= 14.36.
    At the actuator pin, F_x= 39.227 and F_z= -33.503.

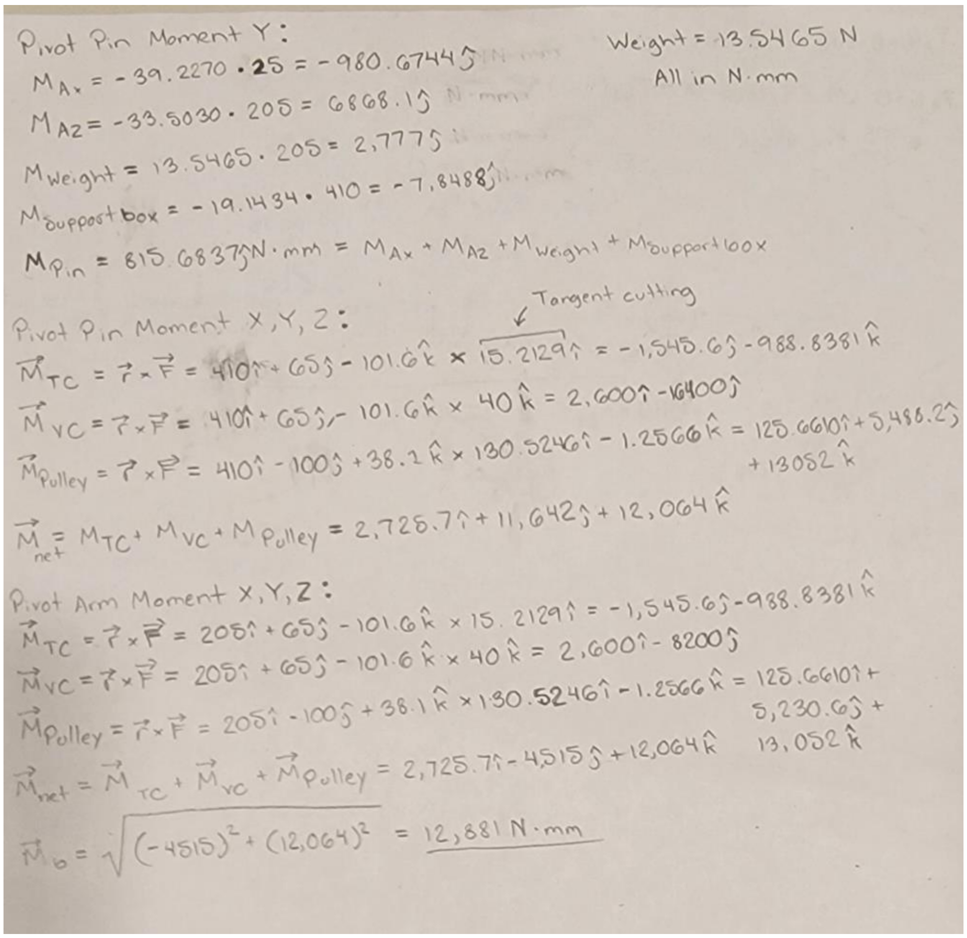
6. What is the magnitude of the peak axial load, the peak bending moment in each direction, and the peak torque on the pivot arm, and where does each occur?

Peak Axial: 146.98N acts from anywhere between the actuator pin and the end of the pivot arm.

Work:



Peak Bending Moment & Torque: 12881 Nmm at the center of the actuator pin.
Work:



Pivot Pin Moment Y:

Weight = 13.5465 N

All in N·mm

$M_{Ax} = -39.2270 \cdot 25 = -980.6744\hat{j}$ N·mm

$M_{Az} = -33.5030 \cdot 205 = 6868.1\hat{j}$ N·mm

$M_{weight} = 13.5465 \cdot 205 = 2{,}777\hat{j}$

$M_{support\ box} = -19.1434 \cdot 410 = -7{,}8488\hat{j}$ N·mm

$M_{Pin} = 815.6837\hat{j}$ N·mm $= M_{Ax} + M_{Az} + M_{weight} + M_{support\ box}$

Pivot Pin Moment X, Y, Z:

Tangent cutting

$\vec{M}_{TC} = \vec{r} \times \vec{F} = 410\hat{i} + 65\hat{j} - 101.6\hat{k} \times 15.2129\hat{i} = -1{,}545.6\hat{j} - 988.8381\hat{k}$

$\vec{M}_{VC} = \vec{r} \times \vec{F} = 410\hat{i} + 65\hat{j} - 101.6\hat{k} \times 40\hat{k} = 2{,}600\hat{i} - 16400\hat{j}$

$\vec{M}_{Pulley} = \vec{r} \times \vec{F} = 410\hat{i} - 100\hat{j} + 38.2\hat{k} \times 130.5246\hat{i} - 1.2566\hat{k} = 125.6610\hat{i} + 5{,}486.2\hat{j} + 13052\hat{k}$

$\vec{M}_{net} = M_{TC} + M_{VC} + M_{Pulley} = 2{,}725.7\hat{i} + 11{,}642\hat{j} + 12{,}064\hat{k}$

Pivot Arm Moment X, Y, Z:

$\vec{M}_{TC} = \vec{r} \times \vec{F} = 205\hat{i} + 65\hat{j} - 101.6\hat{k} \times 15.2129\hat{i} = -1{,}545.6\hat{j} - 988.8381\hat{k}$

$\vec{M}_{VC} = \vec{r} \times \vec{F} = 205\hat{i} + 65\hat{j} - 101.6\hat{k} \times 40\hat{k} = 2{,}600\hat{i} - 8200\hat{j}$

$\vec{M}_{Pulley} = \vec{r} \times \vec{F} = 205\hat{i} - 100\hat{j} + 38.1\hat{k} \times 130.5246\hat{i} - 1.2566\hat{k} = 125.6610\hat{i} + 5{,}230.6\hat{j} + 13{,}052\hat{k}$

$\vec{M}_{net} = \vec{M}_{TC} + \vec{M}_{VC} + \vec{M}_{Pulley} = 2{,}725.7\hat{i} - 4{,}515\hat{j} + 12{,}064\hat{k}$

$\vec{M}_b = \sqrt{(-4515)^2 + (12{,}064)^2} = 12{,}881$ N·mm

DESIGN OF THE PINS

1. Select an appropriate pin that would be suitable for use at all of the pinned connections. That is, the pins must be identical. Justify your selection with appropriate calculations. Provide a McMaster-Carr part number and cost for the pins.

   98381A639 is the part number, and each pin costs $8.02

2. Specify the hole diameter and tolerance you require for these pins. NOTE: depending on your experience in your CAD class, you might need additional information on holes and tolerances. You are expected to use your resources to make an appropriate design choice and to cite your sources.

   The hole diameter is 9.53 mm, which is 0.07 mm larger than the pin because this is the maximum error of this tolerance as given by McMaster
   The tolerance required for the pins is +/- 0.07 mm for a clearance fit to ensure that there is no interference, even if both the hole is smaller than nominal, and the pin is larger. This is assuming that the hole would have the same tolerance error as the pin, because there is no tolerance error provided for this process.

DESIGN OF THE PIVOT ARM

1. Design the pivot arm, choosing your cross-section to meet the strength and deflection requirements. In your design, you may ignore any possible stiffening or strengthening effects of the support box.

```
% Deflection about y and z
def_y = (net_moment_pin(2)*(origin_shaft_length_x)^2)/(2*(69000)*(MoI_arm));
def_z = (net_moment_pin(3)*(origin_shaft_length_x)^2)/(2*(69000)*(MoI_arm));

% Deflection magnitude
def_mag = sqrt(def_y^2+def_z^2);
```

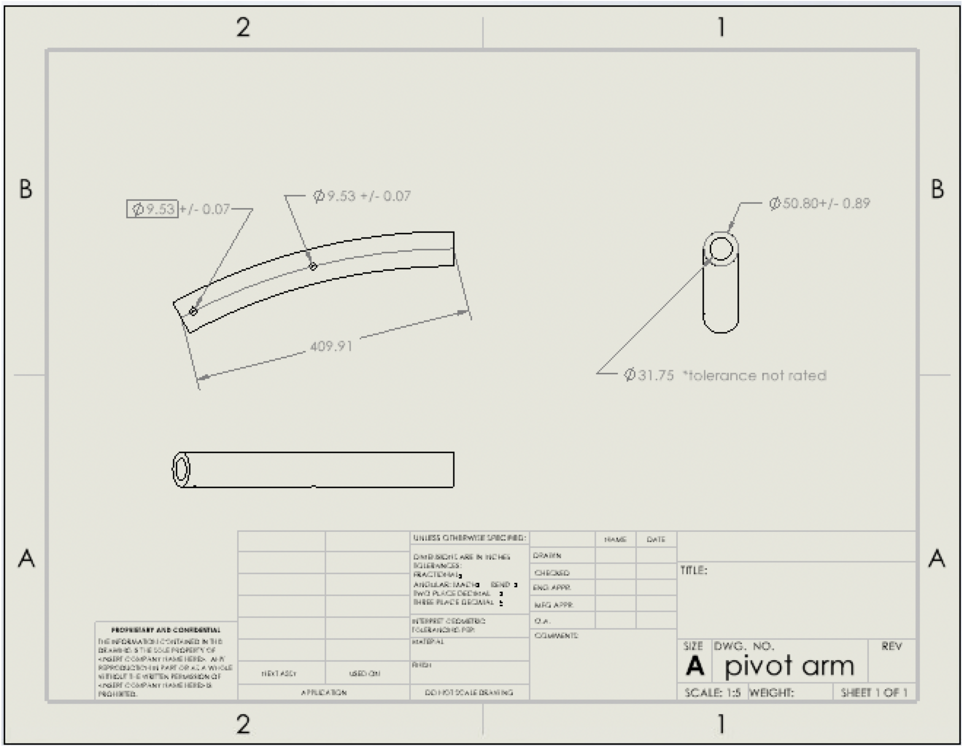
2. Show calculations to justify the strength performance in terms of the local stresses at the pin connections, the overall axial, bending and torsional stresses, and any other important effects. NOTE: as a new engineer, it is unlikely that you ever studied bending in curved beams. You are expected to use your resources to make this calculation and to cite your sources.

```
torsional_shear_pin = (net_moment_pin(1) / polar_MoI_pin)*(pin_r);
% Bending Moment Pin
bending_moment_pin = sqrt(net_moment_pin(2)^2 + net_moment_pin(3));
% Maximum Bending Stress for Pin
bending_stress_pin = (bending_moment_pin * pin_r) / (MoI_pin);
% Von Mises in Pin
von_mises_pin = sqrt(bending_stress_pin^2+3*torsional_shear_pin^2);
```

3. Show calculations to justify the performance in terms of the deflection of the blade shaft support point and the twist angle of the pivot arm. NOTE: for purposes of this initial deflection calculation, you may make the approximation that the pivot arm is straight, rather than curved.

   Deflection in the y direction 0.0005mm
   Total deflection 0.0069mm

4. Produce a CAD drawing of the pivot arm, including dimensions and tolerances, suit- able for quote/fabrication and appropriate for official documentation.



5. Provide a McMaster-Carr part number for your selected cross-section. Your selection must be the shortest available part that is still long enough to use in making the curved pivot arm. Give the cost for that part number (do not reduce the cost if your length is shorter than the stock piece).

    The McMaster Carr part number for the cross section is 9056K14. The cost is $92.

ECONOMICAL COMPETITIVENESS

Provide a summary table for your bill of materials, including the total cost of materials for the pivot arm and the pins. Since everyone will have to bend the pivot arm and drill nearly the same holes, you do not have to provide a machining estimate.

Dwg No.	Part No.	Qty.	Description	Cost/Unit	**Total**
Pivot arm	9056K14	1	Multipurpose 6061 Aluminum Round Tube, 3/8" Wall Thickness, 2" OD	$92	$92
Pins	98381A639	2	4140 Alloy Steel, 3/8" Diameter, 3-3/4" Long	$8.02	$16.04
				Total	$108.04

Parts are sourced from McMaster Carr.

Matlab Code:

```
% Force: N
% Dimension: mm

r_o = 50.8;
d_o = r_o * 2;
t = 9.525;
r_i = r_o - t;
d_i = r_i * 2;
area = pi*(r_o^2-r_i^2);

pin_D = 9.525;
pin_r = pin_D / 2;
MoI_pin = (pi*pin_D^4) / 64;
MoI_arm = (pi/64)*(d_o^4 - d_i^4);

% Polar Moment of Inertia of Pin
polar_MoI_pin = (pi*pin_D^4)/32;

% Polar Moment of Inertia of Pivot Arm
polar_MoI = (pi/32)*(d_o^4 - d_i^4);

w_pivot_arm = 13.5465;

length_shaft = 165;

torque_blade = 1545.63;
torque_motor = 2060.84;
origin_shaft_length_x = 410;
arc_length = 414.5;

r_c = 832.625;
r_n = r_o^2 / (2 * (r_c - sqrt(r_c^2 - r_o^2)));
e = r_c - r_n;
```

```
origin_actuator_length_x = 205;
origin_mounting_length_z = 150;
centershaft_blade_length_y = 65;
centershaft_pulley_length_y = 100;
origin_actuator_length_z = 25;

driving_pulley_radius = 50.8;
driven_pulley_radius = 38.1;
blade_radius = 101.6;

w_support_box = -19.6;
f_cutting_z = 40;

diff_driving_driven_pulley_radius = driving_pulley_radius - driven_pulley_radius;

% Angle of the belt with respect to the origin
angle_pulley = atand(diff_driving_driven_pulley_radius / origin_shaft_length_x);

f_slack_mag = 45;
f_slack_vec = [f_slack_mag*cosd(angle_pulley), 0, f_slack_mag*sind(angle_pulley)];

% Angle of pulley with respect to global axis
f_pulley_tension_diff_mag = (f_slack_mag*driving_pulley_radius*cosd(angle_pulley) + torque_motor) / (driving_pulley_radius * cosd(angle_pulley));
f_pulley_tension_diff_vec = [f_pulley_tension_diff_mag*cosd(angle_pulley), 0, -f_pulley_tension_diff_mag*sind(angle_pulley)];

% Belt force acts in +x direction -z direction
f_belt = f_slack_vec + f_pulley_tension_diff_vec;

% % Reaction force on blade from material acts in +x direction
% Found by taking moment about shaft to yield cutting reaction force
% **Might need to include moments about pulley
f_reaction_blade_x = (torque_blade / blade_radius);

% Force x on shaft w/o cutting reaction force
f_shaft(1) = f_belt(1) + f_reaction_blade_x;

% Force z on shaft from force by pulley, cutting force, and support box
% weight.
f_shaft(3) = f_belt(3) + f_cutting_z + w_support_box;

% Moment on shaft
moment_shaft = [f_cutting_z*centershaft_blade_length_y + f_belt(3)*centershaft_pulley_length_y, 0, f_belt(1)*centershaft_pulley_length_y + f_reaction_blade_x*centershaft_blade_

% Actuator forces (sum of moment about pivot pin eqn)
angle_rotated_pivotarm_tangent_x = atand(origin_actuator_length_z/origin_actuator_length_x);
angle_actuator = 40.5;
f_actuator_mag = (-f_shaft(3)*origin_shaft_length_x)/(sind(angle_actuator)*origin_actuator_length_x + cosd(angle_actuator)*origin_actuator_length_z);
f_actuator_vec = [f_actuator_mag*cosd(angle_actuator), 0, f_actuator_mag*sind(angle_actuator)];

% Pivot forces (sum of forces from actuator, shaft, weight of pivot arm)
f_pivot_vec = [-f_shaft(1) + (-f_actuator_vec(1)), 0, -f_shaft(3) + (-f_actuator_vec(3))];

% Pivot Pin Moment y
m_Ax = f_actuator_vec(1)*origin_actuator_length_z;
m_Az = -f_actuator_vec(3)*origin_actuator_length_x;
m_weight = w_pivot_arm*origin_actuator_length_x;
m_Sz = -f_shaft(3)*origin_shaft_length_x;
moment_pin_arm = [0, m_Ax + m_Az + m_weight + m_Sz, 0];

% Moments Acting on Pivot Pin
m_tangent_cutting_force = cross([origin_shaft_length_x, centershaft_blade_length_y, -blade_radius], [f_reaction_blade_x, 0, 0]);
m_vertical_cutting_force = cross([origin_shaft_length_x, centershaft_blade_length_y, -blade_radius], [0, 0, f_cutting_z]);
m_pulley = cross([origin_shaft_length_x, -centershaft_pulley_length_y, driven_pulley_radius], f_belt);
```

```
% Net Moment in Pin
net_moment_pin = moment_pin_arm + m_tangent_cutting_force + m_vertical_cutting_force + m_pulley;

% Peak Axial Load
f_peak_axial = 146.987;

% Moments Acting on Pivot Arm
m_tangent_cutting_force_pivot_arm = cross([origin_actuator_length_x, centershaft_blade_length_y, -blade_radius], [f_reaction_blade_x, 0, 0]);
m_vertical_cutting_force_pivot_arm = cross([origin_actuator_length_x, centershaft_blade_length_y, -blade_radius], [0, 0, f_cutting_z]);
m_pulley_pivot_arm = cross([origin_actuator_length_x, -centershaft_pulley_length_y, driven_pulley_radius], f_belt);

% Net Moment in Pivot Arm
net_moment_pivot_arm = m_tangent_cutting_force_pivot_arm + m_vertical_cutting_force_pivot_arm + m_pulley_pivot_arm;

% Shear Force in Pivot Arm
shear_force_pivot_actuator = (f_shaft(3)*origin_actuator_length_x)/origin_actuator_length_x;
shear_force_actuator_shaft = w_pivot_arm + f_shaft(3);

c_o = r_o - r_n;

% Bending Moment in Pivot Arm
bending_moment_pivot_arm = sqrt(net_moment_pivot_arm(2)^2 + net_moment_pivot_arm(3)^2);

% Maximum Bending Stress in Pivot Arm
bending_stress_pivot_arm = -(bending_moment_pivot_arm * c_o)/(area*e*r_o);

% Torsional Shear
torsional_shear = (net_moment_pivot_arm(1) / polar_MoI)*(r_o);
% Von Mises in Pivot Arm
von_mises_arm = sqrt(bending_stress_pivot_arm^2+3*torsional_shear^2);

% Torsional Shear for Pin
torsional_shear_pin = (net_moment_pin(1) / polar_MoI_pin)*(pin_r);
% Bending Moment Pin
bending_moment_pin = sqrt(net_moment_pin(2)^2 + net_moment_pin(3));
% Maximum Bending Stress for Pin
bending_stress_pin = (bending_moment_pin * pin_r) / (MoI_pin);
% Von Mises in Pin
von_mises_pin = sqrt(bending_stress_pin^2+3*torsional_shear_pin^2);

% Deflection about y and z
def_y = (net_moment_pin(2)*(origin_shaft_length_x)^2)/(2*(69000)*(MoI_arm));
def_z = (net_moment_pin(3)*(origin_shaft_length_x)^2)/(2*(69000)*(MoI_arm));

% Deflection magnitude
def_mag = sqrt(def_y^2+def_z^2);
```