Design Problem 2 - Chop Saw Blade Shaft Design

Shaft Design Calculations (4 points)

1. *Calculate and report the reaction loads at the bearings (all directions, using the coordinate system defined in Figure 1).*

   Assumptions:

   - Massless bearings – Since we are only concerned with the reaction loads at the bearings, factoring in their mass would introduce contact stresses and other forces that we currently assume to be negligible in relation to the forces acting on the shaft. Additionally, specifications on the bearings aren't provided.
   - Negligible energy loss across the shaft – Since the torque exerted on the shaft by the blade is equal to the torque transmitted to the shaft by the pulley.
   - Bearings are no more than 13mm (1/2") in width – For simplicity, we'll assume the bearings are exactly 13mm in width.

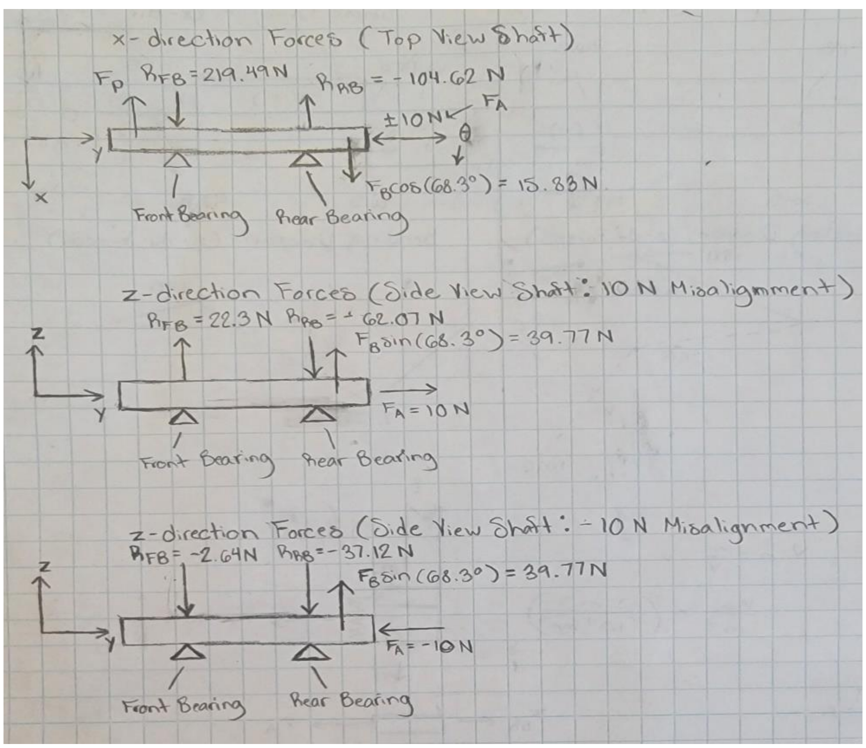

Figure 1: Free-Body Diagram (FBD) of forces acting on shaft and bearings.

Angle θ

Angle $\theta = 111.4°$ with respect to the pulley force acting leftwards, as written in the prompt. When measured from the x-axis, $\theta = 68.3°$.

Solving for Bearing Reaction Forces

See Figure 2 for shaft dimensions.

Force balance $\sum F_x = 0$ and moment balance $\sum M_z = 0$ equations were taken about the front bearing to yield the values in the x-direction:

Moments summed about z-direction to solve for R_{RBx}:

$$R_{RBx} = -\frac{F_P * d_1 + F_B \cos 68.3° * d_2}{L}$$

Forces taken in x-direction to solve for R_{FBx}:

$$\sum F_x = 0 \rightarrow R_{FBx} = F_B \cos 68.3° + F_P - R_{RBx}$$

Force balance $\sum F_z = 0$ and moment balance $\sum M_x = 0$ equations were taken about the front bearing to yield the values in the z-direction (when $F_A = 10$ N):

Moments summed about x-direction to solve for R_{RBz}:

$$R_{RBz} = -\frac{F_B \sin 68.3° * d_2 - F_A * d_3}{L}$$

Forces taken in z-direction to solve for R_{FBz}:

$$\sum F_z = 0 \rightarrow R_{FBz} = -R_{RBz} - F_B \sin 68.3°$$

Force balance $\sum F_z = 0$ and moment balance $\sum M_x = 0$ equations were taken about the front bearing to yield the values in the z-direction (when $F_A = -10$ N):

Moments summed about x-direction to solve for R_{RBz}:

$$R_{RBz} = -\frac{F_B \sin 68.3° * d_2 + F_A * d_3}{L}$$

Forces taken in z-direction to solve for R_{FBz}:

$$\sum F_z = 0 \rightarrow R_{FBz} = -R_{RBz} - F_B \sin 68.3°$$

Bearing Reaction Forces

Bearing Reaction Loads (when $F_A = 10$ N):

$\vec{F}_{FB} = 219.49\hat{\mathbf{I}} - 10\hat{\mathbf{J}} + 22.3\hat{\mathbf{K}}$ N

$\vec{F}_{RB} = -104.62\hat{\mathbf{I}} - 62.07\hat{\mathbf{K}}$ N

Note: This describes the scenario in which the blade misalignment exerts a positive force to the right, countered by an equal and opposite reaction force from the bearing to the left, facilitated by the retaining ring's contact at the front bearing.

Bearing Reaction Loads (when $F_A = -10$ N):

$\vec{F}_{FB} = 219.49\hat{\mathbf{I}} - 2.64\hat{\mathbf{K}}$ N

$\vec{F}_{RB} = -104.62\hat{\mathbf{I}} + 10\hat{\mathbf{J}} - 37.12\hat{\mathbf{K}}$ N

Note: This describes the scenario in which the blade misalignment exerts a negative force to the left, countered by an equal and opposite reaction force from the bearing to the right, facilitated by the spacer's contact at the rear bearing.

2. *Including any stress concentration effects, where would you expect the highest stress in the shaft?*

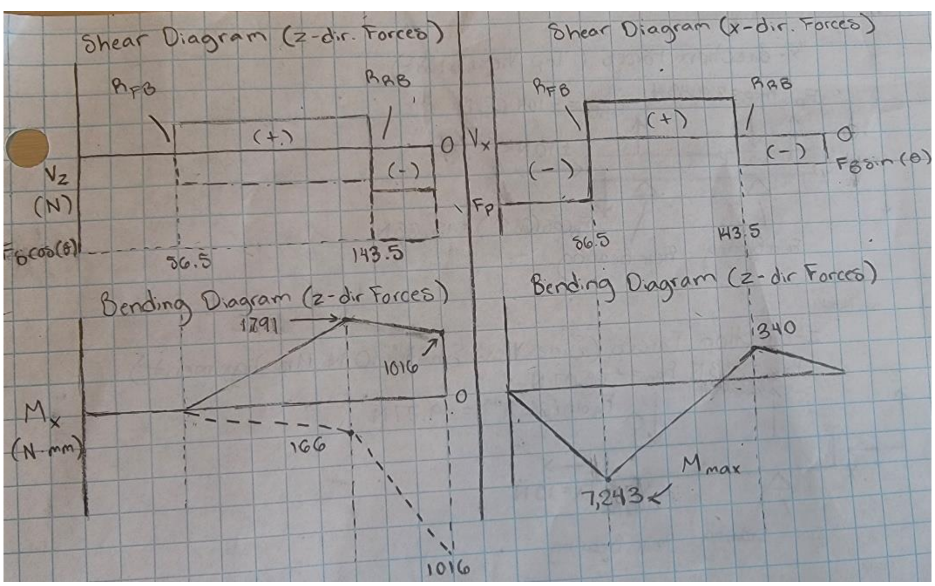

Figure 3: Shear and bending moment diagrams for x and z direction forces.

Using Table A-9 loading condition 10 provided in Shigley from Figure 4 [1, p. 1033] as a guideline, the above shear and bending moment diagrams for both x

and z directions were resolved. This diagram reveals that the maximum moment occurs *at the front bearing*.

Stress concentrations due to the retaining ring groove, however, indicate that highest stress occurs *at the depth of the retaining ring cut-out on the shaft*. However, the bending moment at this location is smaller than at the front bearing. The exact moment value is calculated as follows:

$$M_z = \frac{M_{max}}{d_{pulleyTofrontBearingCenter}} * d_{pulleyToGrooveCutout} = -6401.6 \text{ N} * \text{mm}$$

3. *Determine the total bending moment and torque at the location of highest stress.*

As indicated above by the shear and bending moment diagrams, $M_x = M_y = 0$ at the front bearing and the previously calculated $M_z = -6401.6 \text{ N} * \text{mm}$ bending moment value allows us to determine the total bending moment:

$$M_B = \sqrt{M_x^2 + M_y^2 + M_z^2} = -6401.6 \text{ N} * \text{mm}$$

Given torque transmitted to the shaft from the pulley $T = 1.548 \text{ N} * \text{m}$, torque is constant throughout:

$$T = 1.548 \text{ N} * \text{m} = 1{,}548 \text{ N} * \text{mm}$$

4. *Design the shaft.*

The shaft can only be either *steel* or *stainless steel*.

First Iteration

Selected shaft from McMasterCarr: Rotary Shaft Black-Oxide 1045 Carbon Steel, 1/2" Diameter, 12" Long, Part Number: 1886K21

Factor of Safety:

$n = 1.5$ according to requirements.

Find Brinell Hardness H_B:

Hardness = Rockwell C20 = Brinell 226 (Precision Grinding, Inc., 2023)

Find Ultimate Tensile Strength S_{ut} (which we assume to be the minimum tensile strength to utilize the following equation):

$S_{ut} = 3.4H_B = 768.4$ MPa (Table 2-36, p. 61 Shigley)

Find Theoretical Endurance Limit $S_e{}'$:

$S_e' = 0.5S_{ut} = 384.2$ MPa [Eqn 6-10, p. 305 Shigley] since $S_{ut} \leq 1400$ MPa. Additionally, since the material is carbon steel, our equation $S_e{}' = 0.5S_{ut}$ still applies according to Figure 5 [1, p. 306].

Calculate the Marin Factors and S_e:

$S_e = k_a k_b k_c k_d k_e S_e'$

Surface Factor k_a:

$k_a = aS_{ut}^b = (3.04)(768.4\text{ MPa})^{-0.217} = 0.7190$ The selected values a and b were chosen according to Figure 6 (Equation 6-17 [1, p. 309] and Table 6-2 [1, p. 311])

Size Factor k_b:

$k_b = 1.0$ for an initial guess (Per Shigley's Example [1, Example 7-2])

Loading Factor k_c:

Since it's in torsion and bending, we assume $k_c = 1$ ([1, p. 314]). Remains constant across all iterations.

Temperature Factor k_d:

Loaded at room temperature, so $k_d = 1$ ([1, pp. 315-316])

Reliability Factor k_e:

The highest provided $k_e = 0.702$ value from Figure 7 was selected such that only 100 shafts would fail if 10000 were manufactured.

Therefore:

$$S_{e,1} = (0.7190)(1.0)(1)(1)(0.702)(384.2 \text{ MPa}) = 193.90 \text{ MPa}$$

For K_f, K_{fs}:

$K_f = K_t$ and $K_{fs} = K_{ts}$, notch sensitivity not considered.

For initial diameter, use $K_t = 5$ and $K_{ts} = 3$ per Figure 8 [1, p. 386]

First Iteration Diameter Equation 7-8 [1, p. 382]:

$$d_1 = \sqrt[3]{\frac{16*1.5}{\pi}\left(\frac{2*5*6401.6}{193.9} + \frac{\sqrt{3}*3*1548}{768.4}\right)} = 13.7605 \text{ mm} = 0.54 \text{ in}$$

Nearest standard size: $\frac{1}{2}$ in $= 13.2$ mm

New k_b that aligns with given dimensions using Equations 6-19 [1, p. 312]:

$$k_{b,2} = \left(\frac{d_1}{7.62}\right)^{-0.107} = 0.9387$$

Corrected $S_{e,2}$:

$$S_{e,2} = (0.7190)(0.9387)(1)(1)(0.702)(384.2 \text{ MPa}) = 182.0249 \text{ MPa}$$

The following trye K_f, K_{fs} values were yielded by consulting Figures 9 and 10:

$K_f = 3.2$

$K_{fs} = 2$

…given the specification sheet by fastenermart provided in the bibliography [3] for the design of the retaining ring, the following values were selected for each dimension:

$r = 0.005$ in

$a = 0.039$ in

$t = 0.016$ in

Groove Diameter: 14.9352 mm

Using Equations 7-2 to 7-8 [1, p. 381] and the left term coming from the equation on Figure 9:

$$\sigma_a = K_f \frac{32M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32M_m}{\pi d^3} \tag{7–2}$$

$$\tau_a = K_{fs} \frac{16T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16T_m}{\pi d^3} \tag{7–3}$$

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_a}{\pi d^3}\right)^2\right]^{1/2} \tag{7–4}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2} \tag{7–5}$$

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$$n = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} \tag{7–7}$$

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]^{1/3} \tag{7–8}$$

$$\sigma_{a1} = \frac{32*3.2*6401.6}{\pi*14.9352^3} = 62.7216 \text{ MPa}$$

$$\tau_m = 3*\frac{16*3*1548}{\pi*14.9352^3} = 67.2043 \text{ MPa}$$

$$\sigma_{a2} = 3.2*\frac{4*10}{\pi*14.9352^2} = 0.0334 \text{ MPa}$$

Note that $M_m = 0$ as well as $T_a = 0$

$$n = \left(\frac{\sqrt{\sigma_{a1}+\sigma_{a2}}}{S_e} + \frac{\tau_m}{S_{ut}}\right)^{-1} = 2.9928$$

Passes fatigue criteria.

Second Iteration

$$d_2 = \sqrt[3]{\frac{16*2.9928}{\pi}\left(\frac{2*3.2*6401.6}{182.02} + \frac{\sqrt{3}*2*1548}{768.4}\right)} = 15.2434 \text{ mm} = 0.6 \text{ in}$$

Next standard size up is $\frac{5}{8} in = 15.875$ mm

New shaft from McMasterCarr: Rotary Shaft Black-Oxide 1045 Carbon Steel, 5/8" Diameter, 12" Long, Part Number: 1886K31

Static Fatigue Criteria:

Ignore negligible axial force acting on the groove.

Bending Stress [1, p. 112]:

$I = \frac{\pi}{64} d^4$ (Moment of Inertia for Circle [1, p.1053])

$$\sigma_{bending} = K_t \frac{Mc}{I} = 3.2 * \frac{(6401 \text{ N}*\text{mm})(7.9375 \text{ mm})}{3117.631 \text{ mm}^4} = 52.16 \text{ MPa}$$

Axial Stress [1, p. 123]:

$J = \frac{\pi}{32} d^4$ (Polar Moment of Inertia for Circle [1, p. 1053])

$$\tau_{max} = \frac{Tr}{J} = \frac{(1548 \text{ N}*\text{mm})(3.96875 \text{ mm})}{389.70399 \text{ mm}^4} = 15.76 \text{ MPa}$$

Von Mises Equation 5-15 [1, p. 251]:

$\sigma_y = 0$, no compressive forces.

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2} = 58.87103 \text{ MPa}$$

Yield Check:

According to Piazza post, S_y for 1045 steel is 406.791 MPa, so

$n\sigma' < S_y$

$1.5 * 58.87 < 406.791$

$88.305 \text{ MPa} < 406.791 \text{ MPa}$

Passes static criteria.

Shaft Connection Hardware Design Calculations (4 points)

5. *Design/Select a rectangular key for transmitting torque to the pulley.*

Use Table 7-6 Shigley and use chosen 5/8 in diameter shaft to select a key of width 3/16 and height 3/16. So:

$w = 4.7625\ mm$

$h = 4.7625\ mm$

Determine S_{ut}:

Using hardness table (Precision Grinding, Inc. 2023) and provided hardness from McMasterCarr of Rockwell B50:

$$S_{ut} = 323\ MPa$$

Determine S_y:

From Piazza post [4], the following equation can be used to determine the yield strength for keys:

$$S_y = 0.8 S_{ut} = 258.4\ \text{MPa}$$

Determine minimum area for the key using Maximum Shear Stress Theory Equation 5-3 in Shigley:

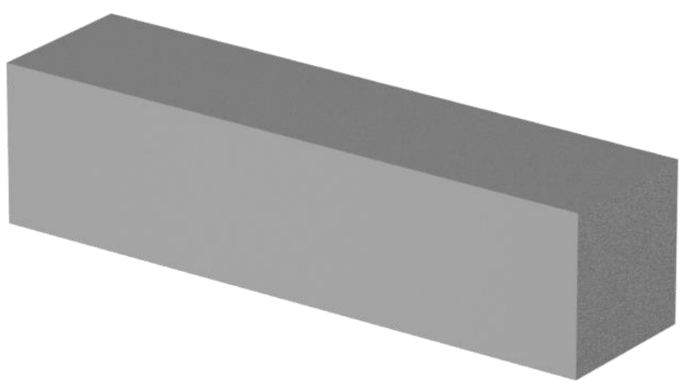
$$A = n * \frac{2T_m}{S_y d} = 1.2 * \frac{2*(1548\ \text{N}*\text{mm})}{(258.4\ \text{MPa})(15.875\ \text{mm})} = 0.9432\ \text{mm}^2$$

Determine minimum length of the key l using known area value from above and using formula to ensure that shear stress dominates:

$$l_{key} = \frac{A}{w} = \frac{0.9432\ \text{mm}^2}{4.7625\ \text{mm}} = 0.198\ \text{mm}$$

$l_{key} > w_{key} \rightarrow 0.198\ \text{mm} > 4.7625\ \text{mm}$ (unsatisfied)

Now, select key from McMasterCarr: Machine Key 1008-1045 Steel, 3/16" x 3/16", 3/4" Long, Undersized, Part Number: 98870A130



Here, $l_{key} = 0.75\ \text{in} = 19.05\ \text{mm}$, now:

$$19.05\ \text{mm} > 4.7625\ \text{mm}$$

Therefore, key dimensions for the shaft are:

$$l_{key}w_{key}h_{key} = 19.05 \text{ mm} * 4.7625 \text{ mm} * 4.7625 \text{ mm}$$

6. *Design/Select an appropriate set screw for securing the pulley and the key axially.*

Table 7. Set Screws for Use Over Keys *ANSI B17.1-1967 (R1998)*

Nom. Shaft Dia.		Nom. Key Width	Set Screw Dia.	Nom. Shaft Dia.		Nom. Key Width	Set Screw Dia.
Over	To (Incl.)			Over	To (Incl.)		
5/16	7/16	3/32	No. 10	2 1/4	2 3/4	5/8	1/2
7/16	9/16	1/8	No. 10	2 3/4	3 1/4	3/4	5/8
9/16	7/8	3/16	1/4	3 1/4	3 3/4	7/8	3/4
7/8	1 1/4	1/4	5/16	3 3/4	4 1/2	1	3/4
1 1/4	1 3/8	5/16	3/8	4 1/2	5 1/2	1 1/4	7/8
1 3/8	1 3/4	3/8	3/8	5 1/2	6 1/2	1 1/2	1
1 3/4	2 1/4	1/2	1/2	…	…	…	…

All dimensions are given in inches.

Determine ss_d:

Using the above page in Machinery's Handbook [5] allows the selection of ss_d =1/4 in

Determine ss_l:

Shigley states that "Setscrews should have a length of about half of the shaft diameter." (1, p. 401). Therefore,

$$ss_l = \frac{d_{shaft}}{2} = \frac{15.875 \text{ mm}}{2} = 7.9375 \text{ mm}$$

Select Set Screw: 18-8 Stainless Steel Cup-Point Set Screw

1/4"-28 Thread, 5/16" Long, Part Number: 92311A557

Table 7-4 (Figure 12) shows the holding power for a ¼ in diameter set screw to be 1000 lbf which approximately equals 4448.22 N. The set screw is not subjected to torsional forces/stresses since it does not resist the torque transmitted from the pulley to the shaft. Removing the set screw does not affect the torque transfer, but instead, it simply keeps the key in place and attached to the pulley. The pulley force imparts 130.7 N or 29.39 lbf to the shaft. The set screw's listed

specifications should be more than required to withstand any potential misalignment force, introducing an axial load on the set screw.

7. *Design/Select the pin for connecting the blade hub to the shaft.*

Force on pin:

$$F_p = \frac{2T}{d_{shaft}} = \frac{2*(1548 \text{ N*mm})}{(15.875 \text{ mm})} = 203.10 \text{ N}$$

$d_{shaft} \gg d_{pin}$ (by a factor of 4), so $d_{pin} \approx 5/32 \text{ in}$

The pin also experiences axial force due to blade misalignment, so the total magnitude force on the pin:

$$F = \sqrt{F_p^2 + F_A^2} = 203.1661 \text{ N}$$

…which has a negligible effect on the magnitude force.

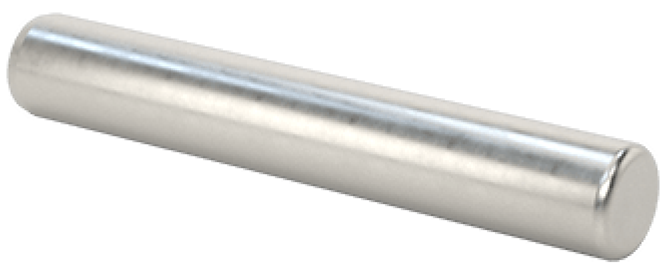
Now calculate the shear area:

$$A_s = \frac{\pi d_{pin}^2}{4} = 11.40 \text{ mm}^2$$

Shear stress on pin with factor of safety $n = 1.2$:

$$\tau_{pin} = n\frac{F}{2*A_s} = 10.68 \text{ MPa}$$

Select pin from McMasterCarr: Dowel Pin 4037, 4140 Alloy Steel, 5/32" Diameter, 1" Long, Part Number: 98381A490



Selected $L = 1 \text{ in} = 25.4 \text{ mm}$

Comparing directly to rated strength listed on McMasterCarr:

$$F_{pin} < Breaking\ Strength \rightarrow 203.16 \text{ N} \ll 4100 \text{ lbf } (18{,}237.71 \text{ N})$$

In order for pin to fail at shear first:

$$h \geq \frac{d_{pin}}{2}$$

where $l_{pin} = 2h + d_{shaft}$

8. *Design/Select a retaining ring for securing the shaft axially.*

The shaft's diameter previously given ($d_{shaft} = 5/8$ in) as well as fastenermart's specifications [3] allows for the selection of the following retaining ring from McMasterCarr: External Retaining Ring for 5/8" OD, Black-Phosphate 1060-1090 Spring Steel, Part Number: 97633A230

…with the following dimensions:

$d_{outer} = 5/8$ in

$t = 0.039$ in thickness to fit within the groove.

The thrust load capacity of this retaining ring is 2090 lbf $= 9{,}296.78$ N, which far exceeds any potentially axial load that might be applied, since the retaining only functions to keep the shaft from sliding rightward. Therefore, there is also no torsional load applied.

Bill of Materials

Part Number	Description	Quantity	Unit Cost
1886K31	Rotary Shaft Black-Oxide 1045 Carbon Steel, 5/8" Diameter, 12" Long	1	$9.00
97633A230	External Retaining Ring for 5/8" OD, Black-Phosphate 1060-1090 Spring Steel	1	$0.15
98381A490	Dowel Pin 4037, 4140 Alloy Steel, 5/32" Diameter, 1" Long	1	$0.89
92311A557	18-8 Stainless Steel Cup-Point Set Screw 1/4"-28 Thread, 5/16" Long	1	$0.12
98870A330	Machine Key 1008-1045 Carbon Steel, 3/16" x 3/16", 3/4" Long, Oversized	1	$0.45
Total Cost			$10.61

Bibliography

[1] R. G. Budynas and J. K. Nisbett, *Shigley's Mechanical Engineering Design*, 11th ed., vol. 1, 1 vols. New York, NY: McGraw- Hill Education, 2020.

[2] *Hardness conversion chart: Precision Grinding, Inc.*. PGI Steel. (2021, September 13). https://pgisteel.com/hardness-conversion-chart/

[3] https://www.fastenermart.com/files/external-retaining-ring-specifications.pdf

[4] https://piazza.com/class/lle9sh08m317nk/post/123

[5] Oberg, E., Jones, F. D., Horton, H. L., & Ryffel, H. H. (2004). *Machinery's Handbook* (27th ed.). Industrial Press.

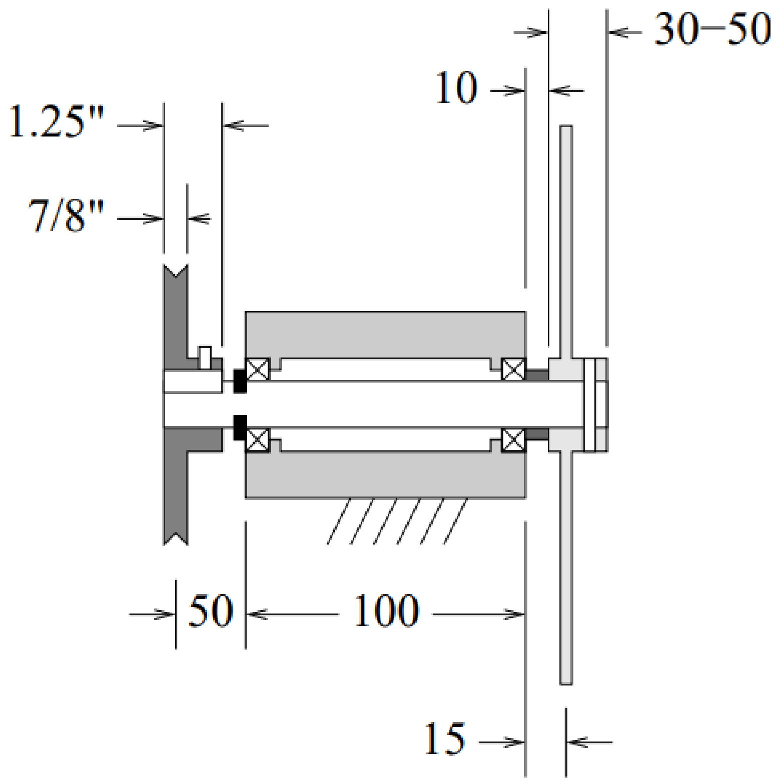


Figure 2: Dimension Sketch (not to scale)

Figure 2: Chop Saw Shaft Mechanism Dimensions

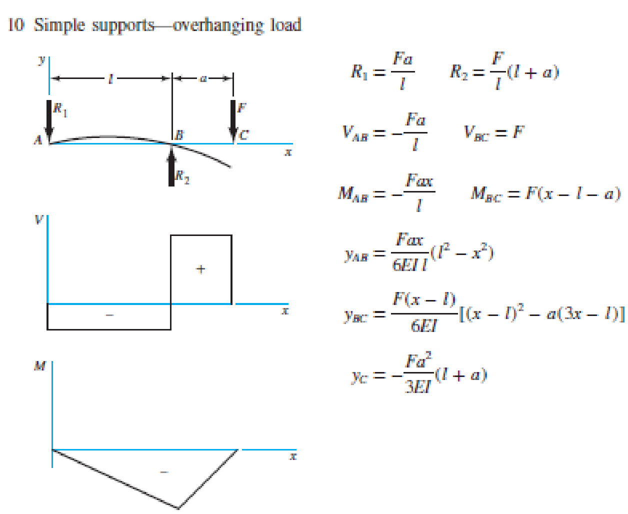


Figure 4: Table A-9-10 from Shigley: Simple supports – Overhanging load [1, p. 1033]

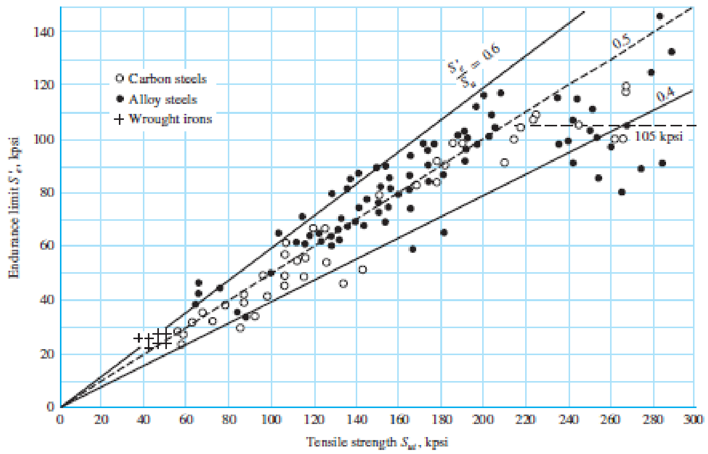


Figure 5: Endurance limits plotted versus tensile strengths for 3 varieties of steel [1, p. 306]

Table 6–2 Curve Fit Parameters for Surface Factor, Equation (6–18)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.21	1.38	−0.067
Machined or cold-drawn	2.00	3.04	−0.217
Hot-rolled	11.0	38.6	−0.650
As-forged	12.7	54.9	−0.758

Figure 6: Table 6-2: Curve Fit Parameter for Surface Factor [1, p. 312]

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702

Figure 7: Reliability Factors k_e Corresponding to 8 Percent Standard [1, p. 317]

	Bending	Torsional	Axial
Shoulder fillet—sharp ($r/d = 0.02$)	2.7	2.2	3.0
Shoulder fillet—well rounded ($r/d = 0.1$)	1.7	1.5	1.9
End-mill keyseat ($r/d = 0.02$)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

Figure 8: First Iteration Estimates for Stress-Concentration Factors K_t and K_{ts} [1, p. 386]

Figure A–15–16

Round shaft with flat-bottom groove in bending and/or tension.

$$\sigma_0 = \frac{4F}{\pi d^2} + \frac{32M}{\pi d^3}$$

Source: W. D. Pilkey, *Peterson's Stress-Concentration Factors*, 2nd ed. John Wiley & Sons, New York, 1997, p. 115.

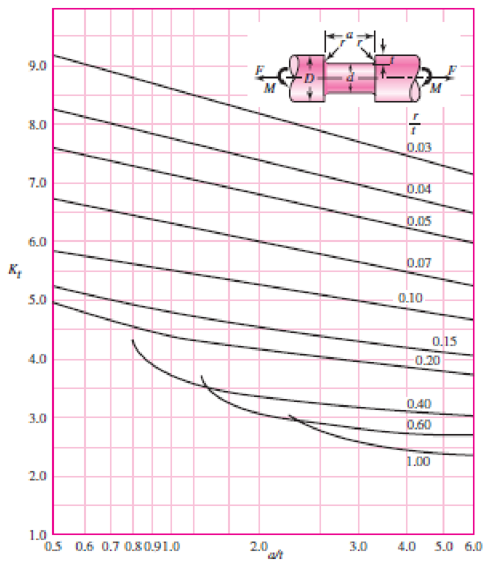


Figure 9: Round shaft with flat-bottom groove in bending and/or tension [1, p. 1047]

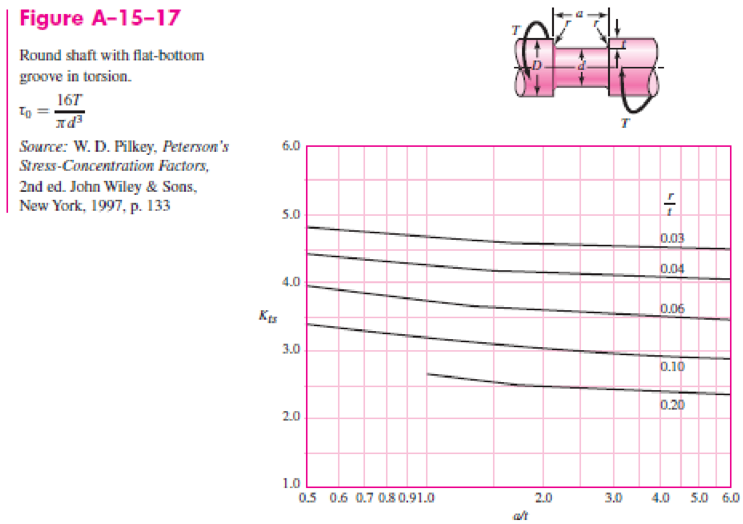


Figure 10: Round shaft with flat-bottom groove in torsion [1, p. 1048]

Table 7–6 **Inch Dimensions for Some Standard Square- and Rectangular-Key Applications**

Shaft Diameter		Key Size		
Over	To (Incl.)	*w*	*h*	Keyway Depth
$\frac{5}{16}$	$\frac{7}{16}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{64}$
$\frac{7}{16}$	$\frac{9}{16}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{3}{64}$
		$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{9}{16}$	$\frac{7}{8}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
		$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{32}$
$\frac{7}{8}$	$1\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{3}{32}$
		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$
$1\frac{1}{4}$	$1\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{1}{8}$
		$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{32}$
$1\frac{3}{8}$	$1\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
		$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{16}$
$1\frac{3}{4}$	$2\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{16}$
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
$2\frac{1}{4}$	$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{7}{16}$	$\frac{7}{32}$
		$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{16}$
$2\frac{3}{4}$	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{8}$

Source: Joseph E. Shigley, "Unthreaded Fasteners," Chapter 24 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design*, 3rd ed., McGraw-Hill, New York, 2004.

Figure 11: Dimensions for Square and Rectangular Key Applications

Table 7–4 Typical Holding Power (Force) for Socket Setscrews*

Size, in	Seating Torque, lbf · in	Holding Power, lbf
#0	1.0	50
#1	1.8	65
#2	1.8	85
#3	5	120
#4	5	160
#5	10	200
#6	10	250
#8	20	385
#10	36	540
$\frac{1}{4}$	87	1000
$\frac{5}{16}$	165	1500
$\frac{3}{8}$	290	2000
$\frac{7}{16}$	430	2500
$\frac{1}{2}$	620	3000
$\frac{9}{16}$	620	3500
$\frac{5}{8}$	1325	4000
$\frac{3}{4}$	2400	5000
$\frac{7}{8}$	5200	6000
1	7200	7000

Source: Unbrako Division, SPS Technologies, Jenkintown, Pa.
*Based on alloy-steel screw against steel shaft, class 3A coarse or fine threads in class 2B holes, and cup-point socket setscrews.

Figure 12: Typical Holding Power (Force) for Socket Setscrews

```
%% Bearing Reaction Forces
dy_faceWidthPulley = 22.225; % mm
dy_supportBox = 100;
theta = 68.3;
F_pulley = [0, -130.7, 0]; % N
%fprintf("Pulley Force: [%di, %.2fj, %dk]\n", F_pulley(1), F_pulley(2), F_pulley(3));
F_mag_blade = 42.8;
%fprintf("Blade Magnitude Force: %.2f N\n", F_mag_blade);
F_blade = [F_mag_blade*cosd(theta), 0, F_mag_blade*sind(theta)];
%fprintf("Blade Vector Force: [%.2fi, %dj, %.2fk]\n", F_blade(1), F_blade(2), F_blade(3));
F_mp = [0, 10, 0];
F_mn = [0, -10, 0];
dy_halfFaceWidthPulley = dy_faceWidthPulley / 2;
dy_supportBox_halfFaceWidthPulley = 50;
dy_widthBearing = 13;
dy_supportBox_blade = 15;
dy_halfWidthBearing = dy_widthBearing / 2;

% Distance from pulley's applied force on shaft to center front bearing
dy_forcePulley_frontBearing = dy_halfWidthBearing + dy_supportBox_halfFaceWidthPulley;
% Distance from belt applied force on shaft to center front bearing
dy_blade_frontBearing = dy_supportBox + dy_supportBox_blade - dy_halfWidthBearing;
% Distance between center of bearings
dy_betweenBearings = dy_supportBox - dy_widthBearing;

% Reaction Forces x - Pulley & Blade
% Sum of moments eqn. about z
Rx_rearBearing = -(F_pulley(2) * -dy_forcePulley_frontBearing + F_blade(1) * dy_blade_frontBearing) / dy_betweenBearings;
%fprintf("Rx - Rear Bearing: %.2f N\n", Rx_rearBearing);
Rx_frontBearing = -Rx_rearBearing - F_pulley(2) - F_blade(1);
%fprintf("Rx - Front Bearing: %.2f N\n", Rx_frontBearing);

% Reaction Forces z - Blade & Misalignment
```

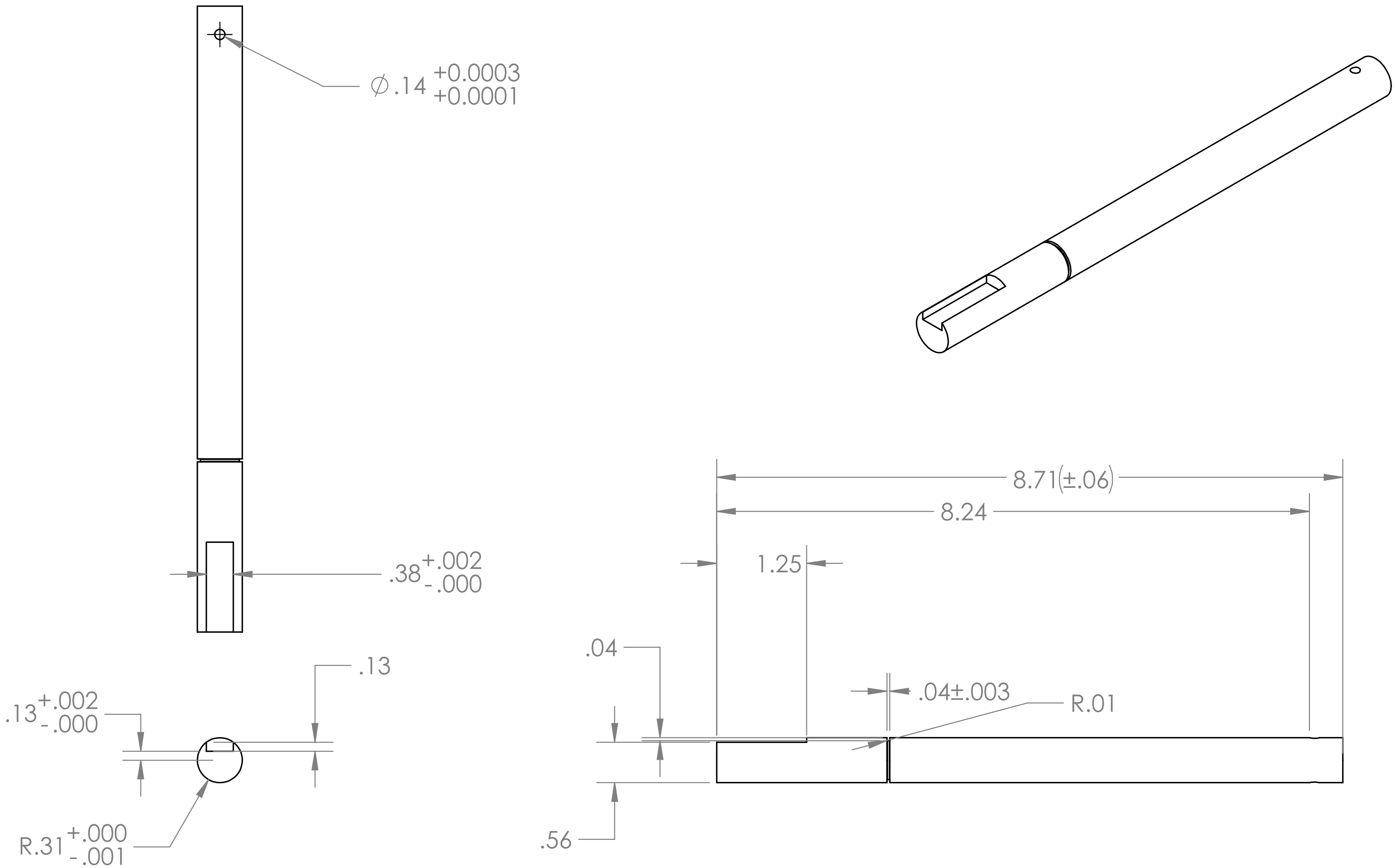
```
% Sum of moments eqn. about x
% Misaligment Force: 10 N
Rzp_rearBearing = (-F_blade(3) * dy_blade_frontBearing - F_mp(2) * dy_blade_frontBearing) / dy_betweenBearings;
%fprintf("Rzp - Rear Bearing (F_M = 10 N): %.2f N\n", Rzp_rearBearing);

Rzp_frontBearing = -Rzp_rearBearing - F_blade(3);
%fprintf("Rzp - Front Bearing (F_M = 10 N): %.2f N\n", Rzp_frontBearing);

% Misaligment Force: -10 N
Rzn_rearBearing = (-F_blade(3) * dy_blade_frontBearing + F_mp(2) * dy_blade_frontBearing) / dy_betweenBearings;
%fprintf("Rzn - Rear Bearing (F_M = -10 N): %.2f N\n", Rzn_rearBearing);

Rzn_frontBearing = -Rzn_rearBearing - F_blade(3);
%fprintf("Rzn - Front Bearing (F_M = -10 N): %.2f N\n", Rzn_frontBearing);

% Reaction Forces y - Blade Misalignment
% When F_M = 10 N, front bearing reaction force on the retaining ring is -10
% N
Ry_rearBearing_1 = 0;
Ry_frontBearing_1 = -10;

%When F_M = -10 N, rear bearing reaction force on the spacer is 10 N
Ry_rearBearing_2 = 10;
Ry_frontBearing_2 = 0;

% Total Reaction Forces - When F_M = 10 N
R_rearBearing_1 = [Rx_rearBearing, Ry_rearBearing_1, Rzp_rearBearing];
R_frontBearing_1 = [Rx_frontBearing, Ry_frontBearing_1, Rzp_frontBearing];
fprintf("Front Bearing when F_M = 10 N: [%.2fi, %.2fj, %.2fk]\n", Rx_frontBearing, Ry_frontBearing_1, Rzp_frontBearing);
fprintf("Rear Bearing when F_M = 10 N: [%.2fi, %.2fj, %.2fk]\n", Rx_rearBearing, Ry_rearBearing_1, Rzp_rearBearing);

% Total Reaction Forces - When F_M = -10 N
R_rearBearing_2 = [Rx_rearBearing, Ry_rearBearing_2, Rzn_rearBearing];
R_frontBearing_2 = [Rx_frontBearing, Ry_frontBearing_2, Rzn_frontBearing];
```

```
fprintf("Front Bearing when F_M = -10 N: [%.2fi, %.2fj, %.2fk]\n", Rx_frontBearing, Ry_frontBearing_2, Rzn_frontBearing);
fprintf("Rear Bearing when F_M = -10 N: [%.2fi, %.2fj, %.2fk]\n", Rx_rearBearing, Ry_rearBearing_2, Rzn_rearBearing);

%% Shaft Design
Mz = (-7243/56.5) * 50-0.889;
T_mm = 1548; % Torque being transmitted
My = T_mm;
Mx = 0; % According to bending moment diagram
FOS_shaft = 1.5;

M_mag = sqrt(Mx^2 + Mz^2); % Bending moments
M_a = M_mag;

hardness_material = 226; % Brinell Scale Hardness

S_ut = 3.4 * hardness_material; % Found in 2-36

% Depends on finish
ka = 3.04 * S_ut^-0.217; % surface factor ka = a*S_ut^b
kb = 1;
kc = 1; % loading factor (remains same)
kd = 1; % temperature factor (remains same)
ke = 0.702; % Reliability of 99.999

S_e_prime = 0.5 * S_ut;
S_e = ka * kb * kc * kd * ke * S_e_prime;

K_f = 5;
K_fs = 3;

shaft_diameter_cubed = (16 * FOS_shaft / pi) * (((2 * K_f * M_a) / S_e) + ((sqrt(3) * K_fs * T_mm) / S_ut));
shaft_diameter = nthroot(shaft_diameter_cubed, 3);
disp(shaft_diameter); % Originally 13.6592 mm
```

```

new_kb = (shaft_diameter/7.62)^-0.107;

%% Retaining Ring Dimensions
% 5/8 (15 mm) OD retaining ring
r_t =  0.005 / 0.016;
a_t = 0.039 / 0.016;
groove_diameter = 0.588 * 25.4;
groove_width = 0.039;

K_f = 3.2; % K_f = K_t
K_fs = 2.0; % K_fs = K_ts

sigma_a_prime = K_f * 32 * (-Mz) / (pi * groove_diameter^3);
tau_m_prime_shear = 3 * (K_fs * T_mm * 16 / (pi * groove_diameter^3))^2;
sigma_m_prime_axial = (K_f * 4 * F_mp(2) / (pi * groove_diameter^2))^2;


FOS_shaft = ((sigma_a_prime / S_e) + (sqrt(tau_m_prime_shear + sigma_m_prime_axial) / S_ut))^-1; % Mod. Goodman
disp(FOS_shaft);

S_e_new = ka * new_kb * kc * kd * ke * S_e_prime;
shaft_diameter_cubed = (16 * FOS_shaft / pi) * (((2 * K_f * M_a) / S_e_new) + ((sqrt(3) * K_fs * T_mm) / S_ut));
shaft_diameter = nthroot(shaft_diameter_cubed, 3);
disp(shaft_diameter);

% Shaft Diameter: 5/8 inch, 14.48 mm computed
% n = 2.6290
% Retaining Ring: 5/8 inch outer diameter
%% Pin

n_comp = 1.2;

F_pin = (2 * T_mm) / shaft_diameter;
```

```matlab
F_pin = (2 * T_mm) / shaft_diameter;

% d_shaft >> d_pin (factor of 4)
% Currently 5/32 inch Dia.
d_pin = shaft_diameter / 4;

F_pin_mag = sqrt(F_pin^2 + (F_mp(2)/2)^2);

area_tau_pin = pi * d_pin^2 / 4;

tau_pin = n_comp * (F_pin_mag / (2 * area_tau_pin));

L_pin = 1 * 25.4; % mm, but adjust in inch

% 4140 Alloy Steel
S_y_pin = 415;

fprintf("h ≥ %.2f mm\n", d_pin);

% Solve for h

%% Key Design

% Maximum Shear Stress Theory
%tau_key_max = S_y_key / (2 * n_comp);

w_key = 3/16 * 25.4;
fprintf("w_key = %.2f should be ≈ %.2f\n", w_key, shaft_diameter / 4);
h_key = 3/16 * 25.4;

S_ut_key = 323; % Found from hardness provided by mcmastercarr
S_y_key = 0.8 * S_ut_key;

```

```matlab
area_key = (2 * n_comp * T_mm) / (shaft_diameter * S_y_key); % Minimum key area
% disp(area_key)

l_key = area_key / w_key; % Minimum key length
fprintf("To ensure shear stress dominates, l_key > %.2f\n", w_key);

% Let l_key be 3/4 in
l_key = 0.75 * 25.4;

%% Set Screw

F_ss = (n_comp * 2 * T_mm) / (shaft_diameter);
disp(F_ss)

% 5/16 inches in length

```



Georgia Institute of Technology	TITLE Shaft		DATE 11/10/2023	NAME Cordell Palmer		
	COURSE ME 3180	SEMESTER FALL 2023	SECTION	LAB	ACTIVITY	DIMENSIONS ARE IN INCHES ANGLES ARE IN DEGREES; SCALE 1:2; SHEET 1 OF 1; SIZE A

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