Design Problem 3 – Blade Shaft Bearings and Gear Design

Gearbox Kinematics, Layout, and Components (2 points)

Assumptions and Requirements:

- Spur gears (requirement 5 in design problem 3 prompt) for both driven and driving function.
- No energy loss across the shaft and gears since torque transmitted remains the same throughout the design.

1. *State the diameter of the blade shaft that you have assumed for your design.*

   It can be observed from McMaster-Carr that for two gears with identical specifications, metric appears to be the least expensive option. Therefore, the shaft's diameter for this design is assumed to be **12 mm (0.47")**, meeting the criteria of exceeding 3/8".

2. *Design the gear train kinematics, stating your final gear reduction ratio (input divided by output) as well as the number of teeth on each gear.*

   The motor generates power within a 3000-3500 RPM speed range, as specified in the design prompt. Calculations below show that a 1.314 gear ratio aligns with the maximum 3500 RPM, and a 1.5333 ratio aligns with the minimum 3000 RPM:

   $$\frac{\omega_2}{\omega_1} = \frac{4600\ RPM}{3500\ RPM} = 1.314$$

   $$\frac{\omega_2}{\omega_1} = \frac{4600\ RPM}{3000\ RPM} = 1.5\overline{3}$$

   …where 4600 RPM is the output angular velocity to be delivered by the blade to the workpiece.

   McMaster-Carr's smallest gear has 12 teeth. To meet the gear ratio range, 16 and 18 teeth gears are ideal due to their lower cost. However, no suitable pitch diameters were found for these combinations.

   Figure 1 indicates the distance from the motor's centerline to the support box edge must be 1.5" (38.1 mm). This requires the driving gear's pitch diameter to be over 3" (76.2 mm), significantly limiting the range of suitable gear ratios for the design.

   From this, a chosen gear ratio of $1.\overline{3}$ was selected to be final gear ratio for the design, with $N_1 = 40$ teeth for the motor gear and $N_2 = 30$ teeth for the shaft gear:

   $$\frac{N_1 = 40}{N_2 = 30} = 1.\overline{3}$$

3. *State the pitch/module and the resulting nominal center distance between the gears.*

   $m = 2$ for the module of both the motor and shaft gears.

   The given equations help determine the pitch diameter (d_p), which is essential for calculating the nominal center distance between gears, as their pitch circles need to be tangent at a specific point:

   $m = \frac{d_p}{N} \rightarrow 2 = \frac{d_p}{40} \rightarrow d_{p1} = 80\ mm$ for the motor gear.

   $m = \frac{d_p}{N} \rightarrow 2 = \frac{d_p}{30} \rightarrow d_{p2} = 60\ mm$ for the shaft gear.

   The nominal center distance is calculated as $r_{p1} + r_{p2}$, the summation of the pitch radius of both meshing gears. Therefore:

   $r_{p_1} + r_{p2} \rightarrow \frac{d_{p1}}{2} + \frac{d_{p2}}{2} \rightarrow \frac{80\ mm}{2} + \frac{60\ mm}{2} = 40\ mm + 30\ mm = 70\ \text{mm}$

4. *Provide the McMaster-Carr part numbers and costs for the two gears.*

   Motor Gear: 2664N25 - $54.71

   Shaft Gear: 2664N24 - $38.69

5. *State the outer diameter for the ball bearings you have selected and the outer diameter you are using for the front and back plates of the support box.*

   The outer diameter (OD) d_b of the selected ball bearings is 28 mm.

   To calculate the OD of the support box's front and back plates d_S, it's necessary to establish the clearance distance between the support box's edge and the bearing's outer diameter, which should be at least 10 mm. The procedure for determining this distance is outlined as follows:

   $r_{p1} + r_{p2} = r_s + w_b + c + 38.1$, where r_s is the shaft radius, w_b the width of the bearing, c clearance.

   Solving for c yields the following equation:

   $c = (r_{p1} + r_{p2}) - 38.1 - w_b - r_s$

   $c = (70) - 38.1 - 8 - 6$

   $c = 17.9\ \text{mm} \geq 10$ (according to the design requirement)

   The front and back plate diameters d_S can be calculated as follows:

   $d_S = 2c + d_b \rightarrow d_s = 2(17.9) + 28$

   $d_S = 63.8\ \text{mm}$

6. *Provide the McMaster-Carr part numbers and costs for the bearings.*
   6001 series, 2RS bearing: 5972K82 - $8.73

Bearing Design Calculations (3 points)

1. *Determine the required static load rating for each of the blade shaft bearings.*

   According to McMaster-Carr, the basic static, radial load capacity C_0 for a 6001-2RS bearing is 530 lbf (2,357 N).

   The provided free-body diagram (FBD) below will be used to assess if the reaction forces on the bearing justify choosing stronger ones.

   - *Note: The bearings are modelled as simple supports to simplify the analysis of the shaft's behavior, since they provide uniform support without significant rotational motion due to proper seating within the housing.
   - Note: Forces are assumed to act at the center of mass of the bearing.

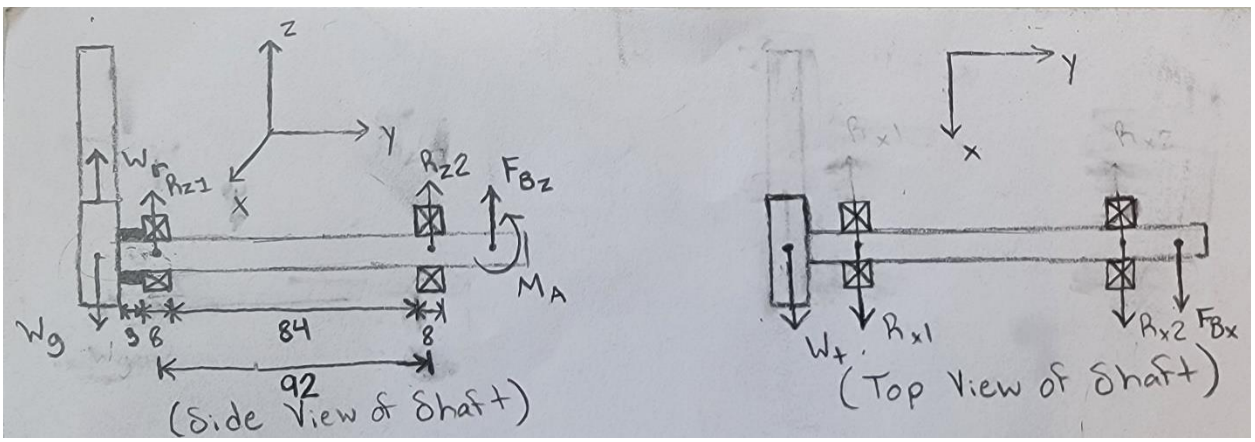


Figure 2 – FBD of the forces acting on the shaft, causing reaction forces on the bearings.

z-Direction Forces ($\sum F_z = 0$):

$W_g = 0.072\ N$ – Weight of the gear acting negative downward on the shaft.

$W_r = 18.78\ N$ – Radial force acting positive upward on the shaft gear.

$F_{Bz} = 39.85\ N$ – Blade force acting positive upward.

R_{z1} & R_{z2} – Reaction forces assumed to act positive upward.

F_A – Axial misalignment force introducing a moment about the front bearing.

$$\sum F_z = 0 \rightarrow R_{z1} + R_{z2} + F_{Bz} + W_r - W_g = 0$$

x-direction Moments ($\sum M_x = 0$):

$$\sum M_x = 0 \rightarrow (R_{z2} * 92) + (F_{Bz})(92 + 4 + 15) + (F_A)(101.6)$$
$$+(W_g)\big(4 + 3 + (15 - 6.1666)\big) - (W_r)(4 + 3 + 10) = 0$$

where:

- 92 mm – Distance between the center of both bearings.
- 4 mm – Distance from the center of a bearing to its edge.
- 15 mm – Distance from the edge of a bearing to the center of the end of the blade (see Figure 1).
- 101.6 mm – Radius of the blade found from the given diameter of 8 inches.
- 3 mm – Provided width of the spacer (see Figure 1).
- $(15 - 6.1666)$ mm – Location of the force which W_g acts based on the material properties found from the SolidWorks file.

Similarly, for the x-direction Forces ($\sum F_x = 0$):

$$\sum F_x = 0 \rightarrow R_{x1} + R_{x2} + F_{Bx} + W_t = 0$$

Similarly, for the z-direction Moments ($\sum M_z = 0$):

$$\sum M_z = 0 \rightarrow -(R_{x2})(92) - (51.6)(4 + 3 + 10) + (15.62)(92 + 15) = 0$$

Therefore, from the above equations, the constant static load experienced by the bearings during operation is as follows:

$$\vec{R}_F = -58.6\hat{\imath} + 10\hat{\jmath} - 2.11\hat{\mathrm{K}}\ \mathrm{N}$$

$$R_F = 59.48\ \mathrm{N}$$

$$R_B = -8.63\hat{\imath} + 10\hat{\jmath} - 55.8\hat{\mathrm{K}}\ \mathrm{N}$$

$$R_B = 57.34\ \mathrm{N}$$

The front bearing experiences a resultant force of 59.48 N, which falls significantly below the static load rating of 2,357 N. The calculated factor of safety is as follows:

$$n = \frac{actual}{required} = \frac{2357\ N}{59.48\ N} = 39.62$$

which greatly exceeds the required factor of safety of $n = 1.2$, passing the static load test.

2. *Determine the required basic dynamic load rating for the blade shaft bearings. Justify your design with complete calculations. No credit will be given without sufficient supporting calculations.*

The desired life of a bearing L_D can be found from equation 11-2b from Shigley, where we need the bearing to spin at 4600 RPM for 20,000 Hrs:

$$L_D = 60 * L * n = 60 * 20000 * 4600 = 5.52e9$$

Equation 11-3 from Shigley is as follows, where $C_{10} = 1100$ lbf (4893 N) sourced from McMaster-Carr:

- $a = 3$ for ball bearings.
- $L_R = 10^6$ as standard life for bearing.

$$C_{10} = F_R = F_D \left(\frac{L_D}{L_R}\right)^{1/a} \rightarrow 4893 = F_D \left(\frac{5.52e9}{10^6}\right)^{1/3} \rightarrow F_D = 276.8 \text{ N}$$

These bearings can support 276.8 N radially for the specified revolutions for 20,000 hours.

The equivalent radial load for ball bearings can be found in figure 2, where it should be noted that $\frac{F_a}{C_0} < 0.014$ is taken as 0.014 according to the table footnote. Additionally, the rotation factor $V = 1$ due to the rotation of the inner ring of the bearing. Equation 11-12 coincides with figure 2 to compute the equivalent radial load. However, to use Equation 11-12 requires first determining whether e exceeds $F_a/(VF_r)$, equation 11-11a:

$$\frac{F_e}{VF_r} = 1 \qquad \text{when} \quad \frac{F_a}{VF_r} \leq e \qquad (11\text{–}11a)$$

$$\frac{F_e}{VF_r} = X + Y\frac{F_a}{VF_r} \qquad \text{when} \quad \frac{F_a}{VF_r} > e \qquad (11\text{–}11b)$$

$$\frac{F_a}{VF_r} \leq e \rightarrow 0.036 \leq 0.19$$

Because equation 11-11a is satisfied, we use $i = 1$.

$$F_e = X_i V F_r + Y_i F_a \rightarrow (1)(1)(276.8) + (0)(10) = F_r = 276.8 \text{ N}$$

Here, we can see the radial load is equal to the equivalent radial load due to negligible axial loads acting on the bearing.

Now we can compute the $n = 1.2$ factor of safety to determine whether the bearing is suitable for radial loads:

$$C_{10} = n * F_D \left(\frac{L_D}{L_R}\right)^{\frac{1}{a}}$$

$C_{10} = 230.71\ \text{N} < 4893\ \text{N}$, which passes the dynamic load rating test by our specified safety factor.

3. *State the minimum thickness for the support box front plate (based only on the bearing and shoulder thickness), the hole size and tolerances for installing the bearing, and the dimensions (thickness, inner diameter, and tolerances) for the through-hole of the shoulder. Cite any sources.*

The support box, made of aluminum 6061 with a yield strength of 240 MPa, experiences only axial forces from the bearings, leading to shear stress. Thus, the effective yield strength is reduced to 120 MPa ($S_y/2$). Additionally, we will assume a default factor of safety of $n = 2$:

$$120 = 2 * \frac{F}{2A}$$

$$\rightarrow 120 = 2 * \frac{10}{2*\pi*r*h}$$

$$\rightarrow 120 = 2 * \frac{10}{2*\pi*r*h}$$

$h = 0.0019$ mm is the minimum shoulder thickness to prevent yielding from axial loading.

The thickness of the shoulder is selected to be 1 mm. Therefore, the overall thickness of the plates is 9 mm.

The bearing has an ABEC Rating of Grade 1, meaning it has looser tolerances. [2] The tolerance stated by McMaster-Carr for a 6001-2RS bearing is -0.009 mm to 0 mm. The hole size for installing the bearing is recommended to be a loose fit for the outer ring, according to SKF [3]. A J7 Housing fit is recommended according to ntnamerica.org [4], based on the following figure:

Table 7.3 (2) Housing fit

Conditions		Housing fits
Static inner ring load	Normal to heavy load	J7
	Normal loads with split housings	H7
Outer ring rotating load	Light loads	M7
	Normal loads	N7
	Heavy and normal loads	P7
Direction indeterminate load	Light loads	J7
	Normal load	K7
	Very heavy or shock load	M7
High demands on running accuracy with light load		K6

Where the static inner ring load condition applies. The same source also lists a table of the recommended tolerances and based on this grade:

Table 7.6 (2) Housing fit

Nominal bore diameter of bearing D (mm)		Δ_{dmp}		G7	H6	H7	J6	J7	Js7	K6
				housing bearing	housing bearing	housing bearing	housing bearing	housing bearing	housing bearing	housing bearing
over	incl.	high	low							
6	10	0	–8	5L~28L	0~17L	0~23L	4T~13L	7T~16L	7.5T~15.5L	7T~10L
10	18	0	–8	6L~32L	0~19L	0~26L	5T~14L	8T~18L	9T~17L	9T~10L
18	30	0	–9	7L~37L	0~22L	0~30L	5T~17L	9T~21L	10.5T~19.5L	11T~11L
30	50	0	–11	9L~45L	0~27L	0~36L	6T~21L	11T~25L	12.5T~23.5L	13T~14L
50	80	0	–13	10L~53L	0~32L	0~43L	6T~26L	12T~31L	15T~28L	15T~17L
80	120	0	–15	12L~62L	0~37L	0~50L	6T~31L	13T~37L	17.5T~32.5L	18T~19L
120	150	0	–18	14L~72L	0~43L	0-~58L	7T~36L	14T~44L	20T~38L	21T~22L
150	180	0	–25	14L~79L	0~50L	0~65L	7T~43L	14T~51L	20T~45L	21T~29L
180	250	0	–30	15L~91L	0~59L	0~76L	7T~52L	16T~60L	23T~53L	24T~35L
250	315	0	–35	17L~104L	0~67L	0~87L	7T~60L	16T~71L	26T~61L	27T~40L
315	400	0	–40	18L~115L	0~76L	0~97L	7T~69L	18T~79L	28.5T~68.5L	29T~47L
400	500	0	–45	20L~128L	0~85	0~108	7T~78L	20T~88L	31.5T~76.5L	32T~53L

1) Applies only to ground shafts.
2) For bearings with negation deviation indicated in bearing tables, same fit applies.
3) T=tight, L=loose, *d*=cone bore, mm, inch
Note: For bearings higher than class 2, consult NTN.

Unit μm
0.0001 inch

Where the nominal bore diameter of the selected bearing is 28 mm. This is in the provide 18-30 mm range in the above table, which suggests that the housing rating be 9T ~ 21L, where T denotes "Tight", L denotes "Loose", and the provided numbers are in units of μm. Therefore, the hole size should be 28 mm with a tolerance of 9 μm tight to 21 μm loose.

The dimensions for the through-hole of the shoulder can be found by citing resource.dynaroll.com, where they state the following: When a bearing is located against a shoulder in a mating part, care must be taken that the rim of the shoulder clears the opposing ring. This is achieved when the diameter of the shoulder has clearance over the opposing race land diameter.

Maximum shaft shoulder diameter = Outer ring land diameter – Clearance*
Minimum housing diameter = Inner ring land diameter + Clearance*
**Clearance should be > 0.010 inch (0.25 mm) to allow for normal tolerances.*" [5]

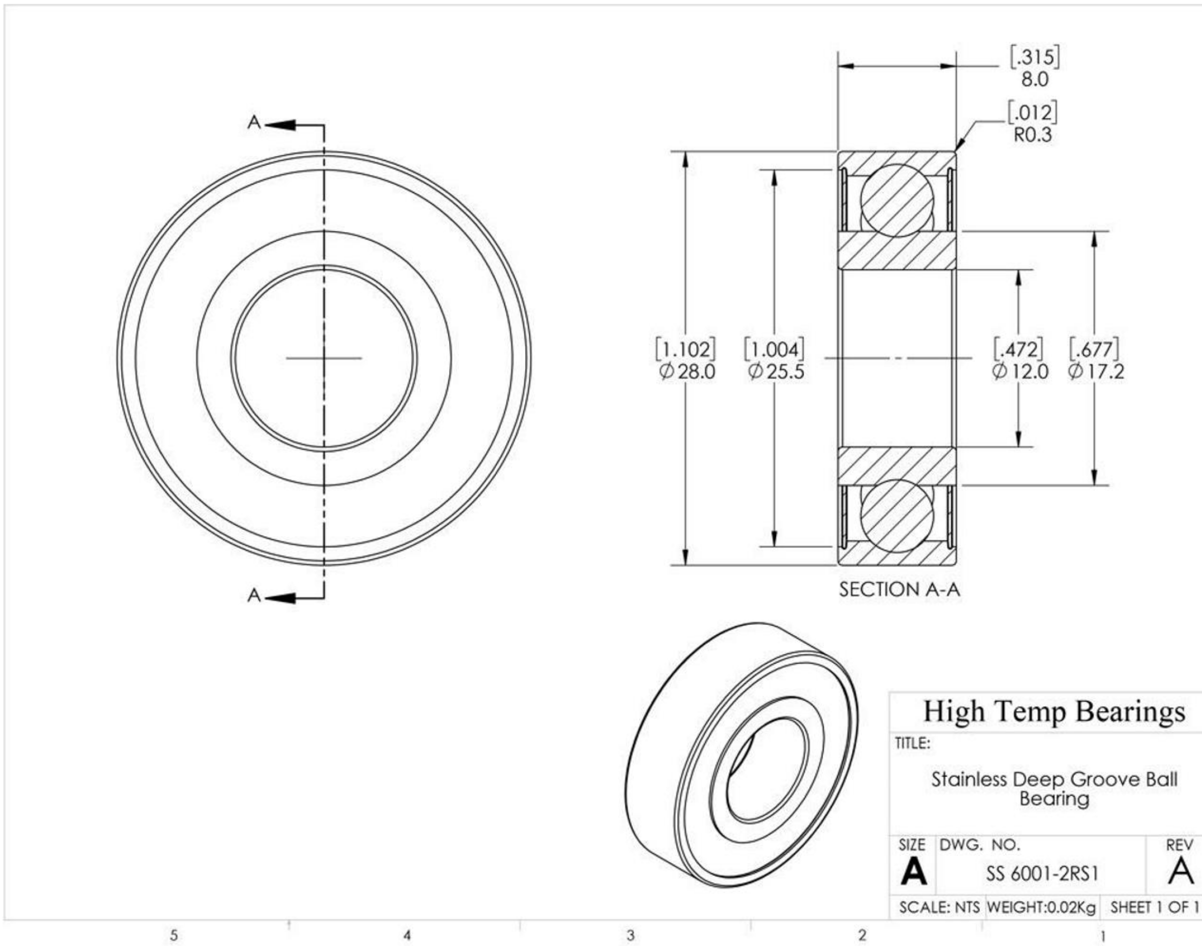
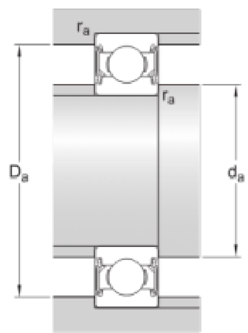


Figure 3 – Dimensions of 6001-2RS1 bearing

The above figure from hightempbearings.com portrays the outer ring, inner diameter (land diameter) to be 25.5 mm. According to the maximum shaft shoulder diameter equation, with a clearance (tolerance) of 0.25 mm, the through hole shoulder is between 25.75 mm and 26 mm. Figure 4 provided below also supports these findings [6].



Abutment dimensions

d_a	min. 0.551 in	Diameter of shaft abutment
d_a	max. 0.591 in	Diameter of shaft abutment
D_a	max. 1.024 in	Diameter of housing abutment
r_a	max. 0.012 in	Radius of shaft or housing fillet

Figure 4 – Abutment dimensions

Additionally, housing radius should be less than bearing radius, or $r > r_a$, where our bearing radius $r = 0.3$ mm. Therefore, we can justify a bearing radius for our housing r_a to be $r_a = 0.15$ mm. [5]

Gear Design Calculations (4 points)

1. *Determine the forces on the gear teeth (radial, tangential, and, if applicable, axial).*

The following equations can be computed by using the gear ratios and torques mentioned in section 1, describing the gear kinematics and torques.

The tangential force acting on the gear teeth can be calculated as follows (Shigley equation 13-12b):

$$W_{t,motor} = \frac{T_{motor}}{\frac{d_{p1}}{2}}$$

where the torque acting on the motor can be calculated as the following, since the blade and shaft gear spin on the same shaft, meaning they possess the same angular velocity and therefore the same torque:

$$T_{shaft} = T_{blade} = 1548 \text{ N*mm}$$

$$T_{motor} = -T_{shaft} * gear\ ratio = -1548 * 1.\overline{3} = -2063 \text{ N*mm}$$

Therefore,

$$W_{t,motor} = \frac{2063 \text{ N*mm}}{\frac{80}{2}\text{ mm}} = 51.6 \text{ N}$$

The radial force on the gear teeth can be calculated as follows (Shigley equation 13-40):

$$W_{r,motor} = W_t \tan\phi, \text{ where } \phi = 20° \text{ pressure angle.}$$

$$W_{r,motor} = 51.6 * \tan 20° = 18.7 \text{ N}$$

It should be noted that the tangential and radial forces act equally and opposite on both gears. Therefore, both gears should experience the same load.

The pitch line velocity for the motor gear is calculated as follows, where $\omega_{motor} = 481.7\ rad/s$, dividing by 1000 yields the answer in m/s, necessary to calculate K_v:

$$V_1 = \left| \frac{\frac{d_{p1}}{2} * \omega_{motor}}{1000} \right| = 14.45 \text{ m/s}$$

The dynamic stress concentration factor K_v is given by the following for hobbed gears (Shigley 14-6c):

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} = 2.068$$

The strength of the selected *steel* gears in bending and contact are provided in the problem statement and are listed as follows:

- $S_t = 210$ MPa
- $S_c = 760$ MPa

The face width for both gears sourced from McMasterCarr are listed as being 20 mm.

$F = 20$ mm

The respective Lewis Form Factors for both gears using table 14-2 in Shigley shows that the motor gear has a $Y = 0.3892$ and the shaft gear has a $Y = 0.359$. Because the shaft gear has a smaller Y, it experiences more stress and must therefore be used as the gear for which all stress calculations are constrained.

The metric Lewis Bending Stress equation (Shigley 14-8) can be used to obtain approximate bending stresses on the gear:

$$\sigma_b = \frac{K_v W^t}{FmY} = 8.3 \text{ MPa}$$

$\sigma_b \ll 210$ MPa, which suggests most stress is not from bending.

The Hertzian Stress (Surface Compressive Stress) can be found using equations 14-11 and 14-12 from Shigley:

$$\sigma_C = -C_p \left[\frac{K_v W^t}{F \cos \phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2} \qquad (14\text{–}14)$$

$$r_1 = \frac{d_P \sin\phi}{2} \qquad r_2 = \frac{d_G \sin\phi}{2} \qquad \text{(14–12)}$$

Where $r_1 = 13.68$ and $r_2 = 10.26$, for the respective pitch diameters of the motor and shaft gear.

And C_p can be found using figure 3:

$C_p = 191$, since our gears are steel.

The Hertzian Stress is therefore equal to:

$\sigma_C = 188 \text{ MPa}$

Though this stress is significantly higher than the bending stress, it falls well below the $S_c = 760 \text{ MPa}$.

To begin the AGMA Gear Stress Analysis, let's first obtain the stress factors and constants needed for both bending and contact:

- Note: All of the following figures from this point are taken from Shigley.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

$K_o = 1.25$ – Reasoning:

Uniform Power Source: If the motor provides a consistent and steady level of power without significant fluctuations, then it would be classified as a uniform power source.

Moderate Shock Driven Machine: A chop saw blade undergoes moderate shock loads because it intermittently cuts through material. The resistance provided by the material being cut can cause sudden and moderate loads on the gear system, especially when encountering knots in wood or unexpected materials in the workpiece.

$$K_s = \frac{1}{k_b} = 1.192 \left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \qquad \text{(a)}$$

$K_s = 1.4045$

$K_H\ (K_m) = C_{mf} = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) = 1.2862$:

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \qquad (14\text{–}31)$$

$C_{mc} = 1$ – The provided schematics and diagrams of the gears listed on McMaster-Carr suggest that the gears are uncrowned.

$$C_{pf} = \begin{cases} \dfrac{F}{10d_p} - 0.025 & F \leq 1 \text{ in} \\ \dfrac{F}{10d_p} - 0.0375 + 0.0125F & 1 < F \leq 17 \text{ in} \\ \dfrac{F}{10d_p} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \leq 40 \text{ in} \end{cases} \qquad (14\text{–}32)$$

Note that for values of $F/(10d_p) < 0.05$, $F/(10d_p) = 0.05$ is used.

$C_{pf} = 0.0250$ – Since $F \leq 1$ in (25.4 mm), the first equation is used. Since the value obtained was less than 0.05, 0.05 is used.

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases} \qquad (14\text{–}33)$$

$C_{pm} = 1.1$ – We must assume the worst case, since the shaft gear follows a cantilever model, not a straddle-mounted one.

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C) \qquad (14\text{–}34)$$

Table 14–9 Empirical Constants A, B, and C for Equation (14–34), Face Width F in Inches*

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

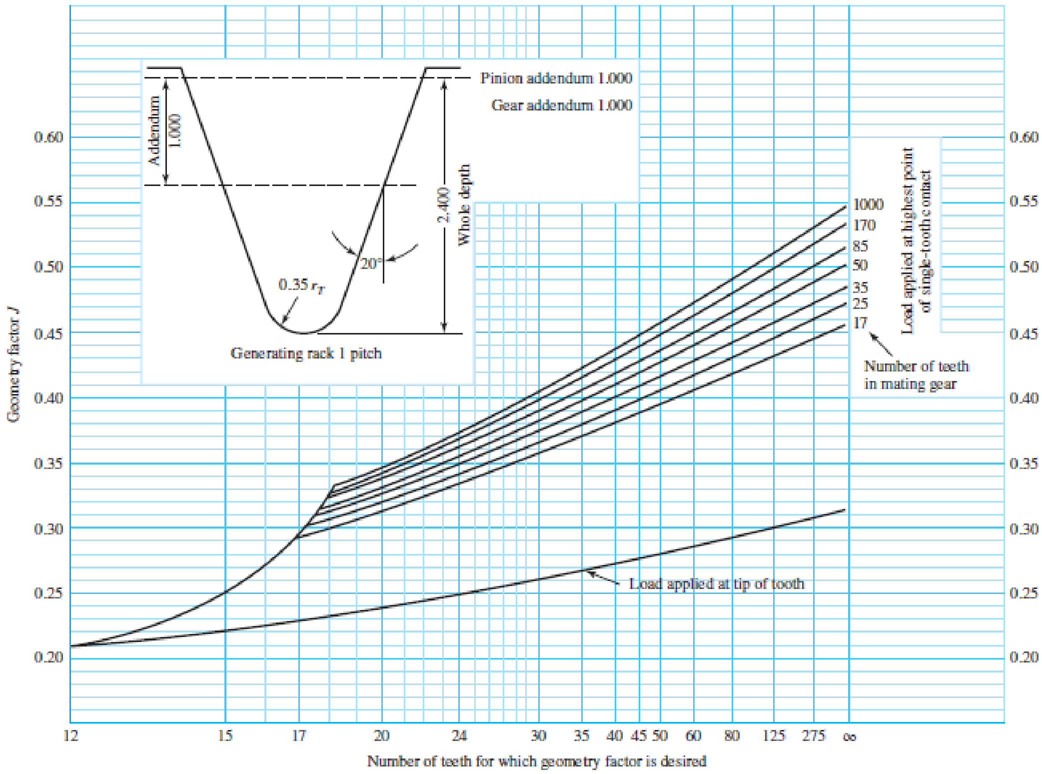
Source: ANSI/AGMA 2001-D04.

$C_{ma} = 0.2587$ - Since the design requires open gearing, the first-row values are used

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

$C_e = 1$ – We assume the gear is not adjusted at assembly and their

material properties remain the same throughout operation and during installation.



$Y_j = 0.37$ – The gear for which we are trying to determine Y_j (shaft gear) has 30 teeth. The driving gear has 40 teeth. We follow the 35 teeth curve to obtain the 0.37 value.

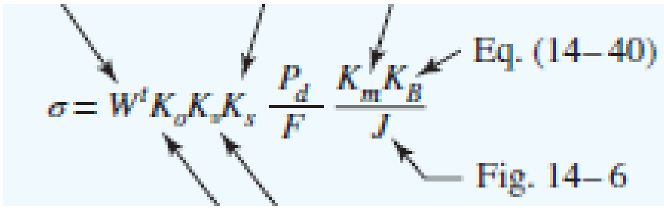
This factor, the *rim-thickness factor* K_B, adjusts the estimated bending stress for the thin-rimmed gear. It is a function of the backup ratio m_B,

$$m_B = \frac{t_R}{h_t} \tag{14–39}$$

where t_R = rim thickness below the tooth, and h_t = the tooth height. The geometry is depicted in Figure 14–16. The rim-thickness factor K_B is given by

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \tag{14–40}$$

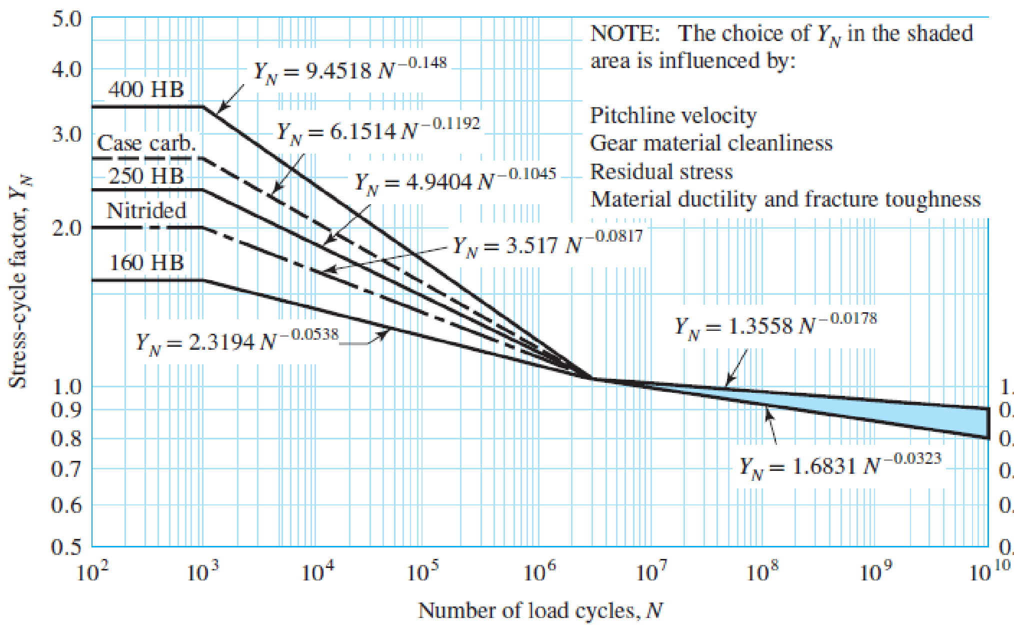
$K_B = 1$ – The rim thickness is much less than the height of the tooth, so we assume that m_B is equal to 1.



We now possess the necessary factors to compute σ_b AGMA Bending Stress in the above figure:

Using the above equation, $\sigma_b = 16.4$ MPa.

To compute the factor of safety, recall that $L = 5.52e9$. We now call this N. With this, we refer to the following figure:



We note that equation $1.3558N^{-0.0178}$ is used, since it was mentioned in lecture, per the AGMA standard, that the upper portion is for general applications.

$Y_N = 0.9095$

$K_T = 1$ – Section 14-15 states that for temperatures up to 250 degrees F, use 1.

Bending factor of safety Eq. (14–41)	$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma}$

Plugging in the above values reveals S_F to be:

$S_F = 11.65$, where the specified minimum safety factor for the gears is 2. It passes the requirement.

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I} \right)^{1/2}$$

To calculate the AGMA contact stress in the above equation, a few additional factors must be found:

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \quad (14\text{–}22)$$

$$m_G = \frac{60}{80} = 0.75$$

$m_N = 1$ – Shigley states on page 758 that this value is 1 for spur gears.

$C_H = 1$ – This value is 1 since both gears are the same hardness and manufactured from the same material.

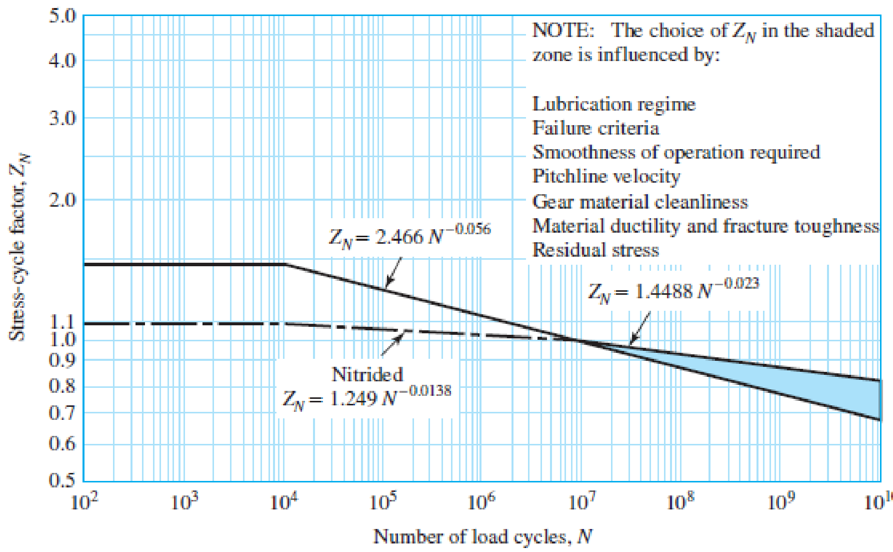
$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G + 1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \quad (14\text{–}23)$$

$I = 0.0689$ – Based on the above obtained values.

Contact stress can now be computed as follows:

$$\sigma_c = 327.26 \text{ MPa}$$

The factor of safety S_H is obtained by calculating Z_N (above figure) and using pre-calculated values:



$Z = 0.8649$

Wear factor of safety Eq. (14–42)	$S_H = \dfrac{S_c Z_N C_H/(K_T K_R)}{\sigma_c}$	Gear only (C_H)

Using the above equation:

$S_H = 2.0072$

2. *If we had a supplier that could provide any face width we wanted at no additional cost, what minimum face width would be necessary for the gears to meet the required factor of safety? Justify your design with complete calculations. No credit will be given without sufficient supporting calculations. Your selected gears must have at least this face width.*

Since the gears experience more stress in contact, we will adjust the iteration according to S_H, not S_F.

Additionally, it should be noted that K_s and K_H are the only factors that scale with the face width F. While K_s scales strictly according to some concrete value of F, care should be taken when proceeding with K_H, since the face width's value depends on the equation to use when calculating C_{pf}. However, we can safely assume that the face width will be no greater than 1 inch since our factor of safety is very roughly 2. Therefore, we can assume that the following equation is always used:

$$\frac{F}{10d_p} - 0.025 \qquad F \leq 1 \text{ in}$$

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

Eq. (14–40)

Fig. 14–6

Using our contact stress equation above, we can use MATLAB to iteratively solve for an F that causes S_H to assume the value of 2.

The procedure to obtain the minimum face width with MATLAB is as follows:

i. Pick a face width.
ii. Calculate S_H
iii. If $S_H > 2.0$, decreases the face width. If $S_h < 2.0$, increase the face width.
iv. Repeat until $S_H = 2.0$

A value of 19.9 mm is found to represent the minimum face width value for the design.

3. *If we wanted to try out a new supplier, what minimum Brinell hardness would you specify for these gears (and why)?*

The S_t and S_c for the steel gears are provided at 99.9% reliability. However, to proceed with the following calculations, they must be readjusted to reflect their 99% reliability, where:

Table 14–10 Reliability Factors K_R (Y_Z)

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

Source: ANSI/AGMA 2001-D04.

Therefore, using the following equations, we can obtain more a standard S_t and S_c:

$$\sigma_{all} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

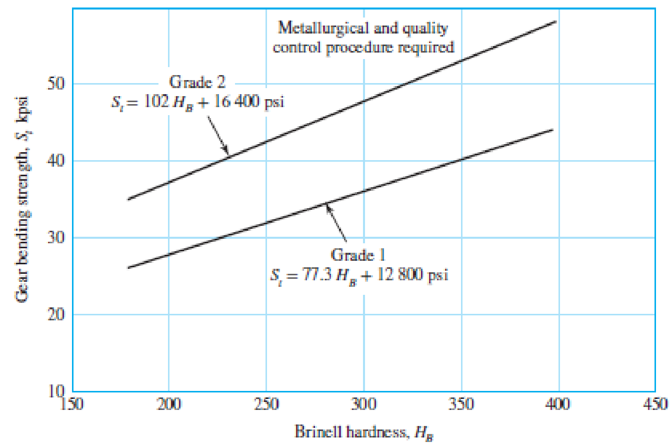
$$\sigma_{c,all} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

Therefore, $S_t = 262.5$ MPa and $S_c = 950$ MPa.

Now, we must take these values and solve for an H_B for both using the following figures:

Figure 14–2

Gear bending strength for through-hardened steels, S_t. The SI equations are: $S_t = 0.533H_B + 88.3$ MPa, grade 1, and $S_t = 0.703H_B + 113$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*



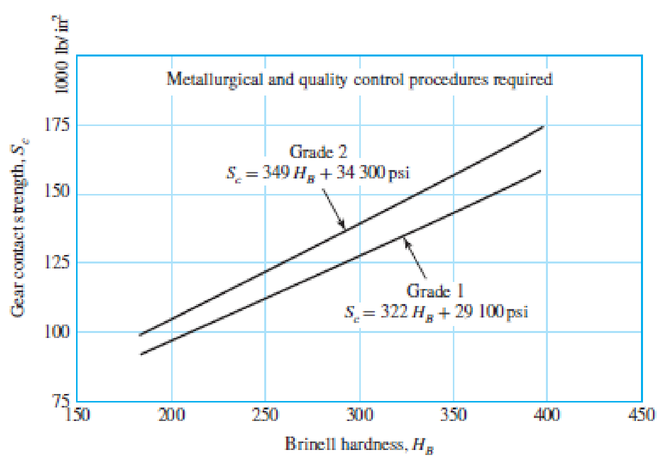


Figure 14–5

Gear contact strength S_c at 10^7 cycles and 0.99 reliability for through-hardened steel gears. The SI equations are: $S_c = 2.22H_B + 200$ MPa, grade 1, and $S_c = 2.41H_B + 237$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

We will use grade 1 steel since we don't know and shall assume worst case.

$S_t = 0.533H_B + 88.3 \rightarrow 262.5 = 0.533H_B + 88.3 \rightarrow H_B = 326$ MPa

$S_c = 2.22H_B + 200 \rightarrow H_B = 337$ MPa, selected since it's larger than 326 MPa.

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Figures

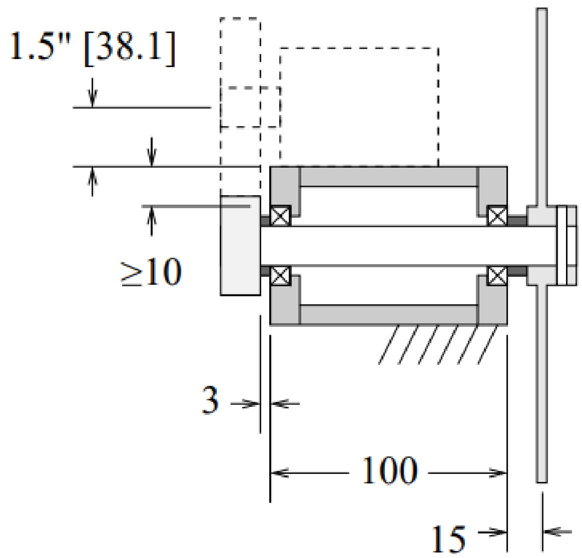

Figure 2: Geared Design Dimensions

Figure 1 – Geared Design Dimensions

Table 11–1 **Equivalent Radial Load Factors for Ball Bearings**

F_a/C_0	e	$F_a/(VF_r) \le e$		$F_a/(VF_r) > e$	
		X_1	Y_1	X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

*Use 0.014 if $F_a/C_0 < 0.014$.

Figure 2 – Equivalent Radial Load Factors for Ball Bearings

Table 14–8 **Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$)**

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in^2 (MPa)*					
		Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

Source: AGMA 218.01

Figure 3 – Elastic Coefficient (C_p)